KRANNERT SCHOOL OF MANAGEMENT

Purdue University
West Lafayette, Indiana

The Benefit of Exchange Rate Flexibility, Trade Openness and Extensive Margin

By

Kanda Naknoi

Paper No. 1215
Date: November, 2008

Institute for Research in the
Behavioral, Economic, and
Management Sciences

The Benefit of Exchange Rate Flexibility, Trade Openness and Extensive Margin

Kanda Naknoi *

Purdue University

November 2008

Abstract

The literature on optimum currency areas argues that in the presence of country-specific real shocks, the cost of fixing exchange rates is decreasing in the degree of trade openness. This study uses a stochastic dynamic general equilibrium model of endogenous specialization to assess the benefit of exchange rate flexibility. The benefit of exchange rate flexibility consists of the benefit along the extensive margin through adjustment in the composition of trade, and the benefit along the intensive margin through adjustment in the relative prices. Openness is found to influence these two benefits differently. Thus, the model predicts a non-monotonic relationship between openness and the benefit of exchange rate flexibility.

JEL Classifications: F41, F42

Keywords: Exchange rate regimes; Trade costs; Openness

*Department of Economics, Purdue University, 403 W. State St., West Lafayette, IN 47907. Email: knaknoi@purdue.edu. I thank David Papell for comments on an earlier version of this paper.

1 Introduction

The major benefit of exchange rate flexibility is to facilitate international relative price adjustment in response to country-specific real shocks in the presence of nominal rigidities (Friedman, 1953). The existing theory argues that the benefit is increasing in the degree of trade openness measured by the trade-to-GDP ratio (Frenkel and Aizenman, 1982; Sutherland, 2006; Kollmann, 2004). This is because a high degree of openness implies that the reallocation of factors of production takes place mostly within the traded-goods sector, not between the traded- and nontraded-goods sectors. Hence, a high degree of openness does not require large adjustments of the internal relative price or the price of nontraded goods relative to traded goods. However, the recent work by Adrian and Gros (2004) argue that the benefit is not monotonically increasing in the degree of openness, because openness increases vulnerability to potentially large external shocks.

This paper contributes to the debate by showing that the benefit of exchange rate flexibility is not monotonic in the degree of trade openness, when exchange rate fluctuations influence the specialization pattern. We evaluate the benefit using the stochastic dynamic general equilibrium (SDGE) model with endogenous specialization and wage rigidities in Naknoi (2008).[1] To simplify the model, trade is assumed to be driven by comparative advantage, as in Dornbusch, Fischer and Samuelson (1977), instead of monopolistic competition, as in Ghironi and Melitz (2005). In the short run, changes in the relative wage cause exporters to switch in and out of exporting and consequently generate endogenous fluctuations in the composition of trade. We refer to such fluctuations as adjustments along the extensive margin. These fluctuations are absent in standard macro models studying adjustments of prices and quantities, or the so-called intensive margin.

This modeling approach is motivated by the evidence for entries and exits among exporters in Besedes and Prusa (2006). Ghironi and Melitz (2005) and Naknoi (2008) use

[1]See Bergin and Corsetti (2008) for recent work on the impact of monetary policy on adjustments along the extensive margin of production in a closed-economy setup.

this approach to explain dynamics of the real exchange rate (RER). Although this study uses the same model as Naknoi (2008), it focuses on different variables and answers an entirely different question. To be precise, we compare the welfare across exchange rate regimes to find a relationship between the benefit of exchange rate flexibility and the degree of trade openness.

Exchange rate regimes are characterized by the interest rate rules. To fix the exchange rate, the home central bank follows the exchange rate targeting rule similar to Benigno (2004) and Monacelli (2004). When the home central bank floats its exchange rate, it follows the feedback rule adopted by Chari, Kehoe and McGrattan (2002) and Naknoi (2008), who found that the feedback rule can replicate the RER dynamics quite well. The foreign central bank follows the same feedback rule regardless of the exchange rate regime. The measure for welfare loss from fixing exchange rates is the permanent decrease in consumption that yields the same expected utility as the flexible exchange rate regime, similar to the measures in Sutherland (2006), Devereux and Engel (2003) and Kollmann (2004).

The key mechanism is that a rise in the relative wage, which is increasing in exchange rate appreciation, causes some firms to quit exporting. These firms have lower productivity than other exporters but higher productivity than other nontraded-goods producers, thus they raise the average productivity in both sectors, and more so in the nontraded sector. Hence, endogenous productivity fluctuations following adjustments along the extensive margin offset movements in the relative wage and reduce volatility of the terms of trade. For this reason, adjustments along the extensive margin have a stabilization effect.

In the quantitative part of the paper, we calibrate the model with country-specific shocks on total factor productivity (TFP). We find that when the two countries have symmetric size, the flexible exchange rate regime yields higher welfare, despite the fact that the interest rule under the flexible regime is not an optimal rule. Thus, an optimal rule would further raise the benefit from floating exchange rates and would not change the cross-regime welfare ranking. The result is also consistent with Devereux and Engel (2003) given that firms practice producer currency pricing.

To quantify the stabilization effect of adjustments along the extensive margin, we decompose the benefit of exchange rate flexibility into the benefit along the extensive margin and the benefit along the intensive margin. Specifically, the benefit along the intensive margin is the welfare loss from fixing exchange rate in the model with exogenous specialization given by the steady state trade pattern in the benchmark model. We measure the benefit along the extensive margin as the difference between the welfare loss in the benchmark model and that in the model with exogenous specialization. The predicted benefit along the extensive margin in the home country is as large as 1 percent of consumption, and equivalent to three times of the benefit along the intensive margin.

Next, we vary the country size and the trade costs parameters to study the effects of openness on the welfare comparison. There are three findings. First, the degree of openness is decreasing in country size and trade costs. When we decompose the degree of openness into the extensive and intensive margins, we find that the extensive margin of openness is increasing in country size, but decreasing in trade costs. This finding is consistent with the evidence in Hummels and Klenow (2005). In our model, the large country exports a wide range of goods because its large labor supply drives down the relative wage. A rise in trade costs reduces the extensive margin, because it causes low productivity exporters to quit exporting. On the other hand, the intensive margin of openness is decreasing in country size but increasing in trade costs. Large labor supply in the large country yields large output for consumption and thus reduces the expenditure share of imports. A rise in trade costs increases the intensive margin of openness because it raises prices for all traded goods.

Second, the benefit from exchange rate flexibility along the extensive margin is increasing in country size, but the benefit along the intensive margin is non-monotonic in country size. Large country reaps large benefit along the extensive margin, because it its large set of goods broadens the scope of reallocation of factors between the traded- and nontraded-goods sectors. The non-monotonicity of the benefit along the intensive margin arises from the fact that adjustments along the intensive margin come from both price and quantity adjustments. Since demand falls when price rises, the effect of price changes on the consumption expenditure is ambiguous. For this reason, the intensive margin adjustment drives the non-monotonicity between the benefit and country size. A small open economy is actually better off with the flexible exchange rate regime, as in Clarida, Gali and Gertler (2002).

Finally, the benefit from exchange rate flexibility along both extensive and intensive margins is non-monotonic in trade costs. The non-monotonicity of the benefit along the extensive margin is due to the fact that a change in trade costs changes the range of export goods in the opposite direction from the range of import goods. Hence, as trade costs rise, volatility of the exchange rate increases adjustments along the extensive margin in the export sector, but decreases those in the import sector. The non-monotonicity of the benefit along the intensive margin comes from the same reason as when we vary the country size. Given the non-monotonicity of the benefit along the two margins, the combined benefit is non-monotonic in trade costs.

To summarize, our model predicts that the benefit from flexible exchange rates is non-monotonic in the degree of openness. The model also predicts that a small open economy is better off with the flexible exchange rate system, as in Clarida et al. (2002). The non-monotonicity is in line with the recent work by Adrian and Gros (2004), who emphasize the role of external shocks.

The adjustments along extensive margin also partially explain why the correlation between relative consumption and the RER is low. Backus and Smith (1993) found that the correlation in the data of OECD countries is 0.05 on average. While the model by Ghironi and Melitz (2005) generates a correlation of 0.71, our predicted correlation is only 0.55. This result is driven by the low correlation between the traded and nontraded

RERs, which results from the offsetting effect of endogenous productivity fluctuations on the terms of trade. The zero correlation reduces the correlation between the traded RER and relative consumption of traded goods, and also the correlation between the nontraded RER and relative consumption of nontraded-goods. Hence, the correlation between relative consumption and the RER becomes low.

The rest of the paper is organized as follows. The next section explains the model. The calibration and results are in Section 3. Section 4 concludes the paper.

2 The model

This section explains the SDGE model of comparative advantage with money also found in Naknoi (2008). Wages are assumed to be sticky in order to study the role of exchange rate regimes. This assumption is motivated by the evidence in Liu and Phaneuf (2007), Kahn (1997), Castellanos et al. (2004) and Huang and Liu (2002). There are two countries: Home and Foreign. There is a continuum of goods indexed by $z \in (0, 1]$. There is a continuum of competitive firms producing goods in each industry. Since the key mechanism is related to the exporting decision of firms, we begin describing the model from the supply side.

2.1 Firms

A large number of homogeneous firms take price as given in each industry z . Let X_t be the total factor productivity, and $a_t(z)$ be the industry-specific productivity. The subscript t denotes the period. Goods prices are fully flexible, hence the invoice currency is irrelevant. Let the producer price, $p_t^p(z)$, be in the seller's currency. The representative firm in each industry produces the output $y_t(z)$ from the labor input $l_t(z)$ with the linear technology,

$$y_t(z) = X_t a_t(z) l_t(z).$$

Let W_t be the unit labor cost. The cost minimization gives the marginal-cost pricing,

$$p_t^p(z) = W_t/(X_t a_t(z)).$$

Similar equations hold for the foreign firms.

2.2 Pattern of specialization

Let $\Phi_{t,a}(z)$ denote the cost of beginning to export. This helps keep the number of new exporters in line with the estimate in Hummels and Klenow (2005). It is an iceberg cost, which reduces productivity and raises price such that $a_t(z) = (1 - \Phi_{t,a}(z))\bar{a}(z)$, where the superscript "$-$" denotes the steady state. The cost is time-variant and increasing in deviations of the long-run relative productivity of the previously least-competitive industry from that of the current one. Define the industry-specific *relative productivity* as $A_t(z) = a_t(z)/a_t^\star(z)$, where the superscript $\star$ denotes the foreign variables. Define the set of new home export industries as $\mathbf{Z}_t^n$, and the set of disappearing home export industries as $\mathbf{Z}_t^d$. z_t^l and z_t^h denote the endogenously-determined least-competitive industry in the home and foreign country, respectively. ϕ_a is a parameter where $\phi_a > 0$. The entry cost for the home producers is given by the following:

$$\Phi_{t,a}(z) = \begin{cases} \phi_a \left[\bar{A}(z_{t-1}^l)/\bar{A}(z_t^l) - 1\right] & for \quad z \in \mathbf{Z}_t^n \cup \mathbf{Z}_t^d \\ 0 & otherwise \end{cases}$$

A similar cost function applies to the foreign producers.

International trade is subject to the iceberg trade costs melting a fraction τ of goods. Define the relative wage as $\omega_t = W_t/S_t W_t^\star$, and the relative TFP as $\chi_t = X_t/X_t^\star$. Dornbusch et al. (1977) showed that if $\partial A_t/\partial z < 0$ and $0 < \tau < 1$, then for any given relative wage ω_t there is a unique solution for z_t^l and z_t^h such that $0 < z_t^l < z_t^h < 1$ and

$$A_t(z_t^l)\chi_t(1 - \tau) = \omega_t = A_t(z_t^h)\chi_t/(1 - \tau). \tag{1}$$

The specialization pattern is as follows. The home economy produces the goods $z \in [0, z_t^h]$

and exports the goods $z \in [0, z_t^l]$. The foreign economy produces the goods $z \in [z_t^l, 1]$ and exports the goods $z \in [z_t^h, 1]$. Both produce the nontraded goods $z \in (z_t^l, z_t^h)$ for domestic consumption.

In the steady state, $\Phi_{t,a} = 0$ for all z. When new home exporters emerge, $z_t^l > z_{t-1}^l$, $\mathbf{Z}_t^d = \emptyset$, and $\Phi_{t,a} > 0$ for $z \in \mathbf{Z}_t^n$. When some home exporters quit, $z_t^l < z_{t-1}^l$, $\mathbf{Z}_t^n = \emptyset$ and $\Phi_{t,a} < 0$ for $z \in \mathbf{Z}_t^d$. There are exit benefits from recoverable value in the industrial organization literature (Ericson and Pakes, 1995). Hence, the entry cost raises the slope of the relative productivity schedule, and productivity of the exporters relative to non-exporters at the margin. The cost reduces the range of switching industries by pushing those about to enter exporting back into the nontraded sector and throwing those about to quit back into exporting. Since the cost is not a fixed cost but is proportional to the relative productivity at the margin, the cost does not remove switching even with small shocks. It creates discontinuity in the relative productivity schedule, but retains the monotonicity along each segment.

Proposition 1 *Suppose $dA_t(z_t)/dz_t < 0$. Then $dz_t^l/d\omega_t < 0$ and $dz_t^h/d\omega_t < 0$ for all t.*

Proof. From (1), $dA_t(z_t^l)/d\omega_t = (dA_t(z_t^l)/dz_t^l)(dz_t^l/d\omega_t) = 1/((1-\tau)\bar{\chi}) > 0$. When $dA_t(z_t)/dz_t < 0$, $dz_t^l/d\omega_t < 0$. Similarly, $dA_t(z_t^h)/d\omega_t = (dA_t(z_t^l)/dz_t^h)(dz_t^l/d\omega_t) = (1-\tau)/\bar{\chi} > 0$. When $dA_t(z_t)/dz_t < 0$, $dz_t^h/d\omega_t < 0$. ■

A rise in the relative wage reduces the comparative advantage of the home country and causes the home exporters at the margin to exit. At the same time the foreign nontraded-goods producers at the margin begin to export. As a result, the range of export goods expands but the range of import goods shrinks.

From the home residents' perspective, we can classify sectors into the import, export and nontraded sectors, and denote them by $i \in (F, H, N)$, respectively. Define $\mathbf{Z}_{t,F} = [z_t^h, 1]$, $\mathbf{Z}_{t,H} = [0, z_t^l]$, and $\mathbf{Z}_{t,N} = (z_t^l, z_t^h)$. The consumer prices in each location are:

$$p_t(z) = \begin{cases} S_t W_t^* / (X_t^* a_t^*(z)(1-\tau)) & for \quad z \in \mathbf{Z}_{t,F} \\ W_t / (X_t a_t(z)) & for \quad z \in \mathbf{Z}_{t,H} \cup \mathbf{Z}_{t,N} \end{cases}$$

$$p_t^*(z) = \begin{cases} W_t^*/(X_t^* a_t^*(z)) & for \quad z \in \mathbf{Z}_{t,F} \cup \mathbf{Z}_{t,N} \\ W_t/(S_t X_t a_t(z)\ (1-\tau)) & for \quad z \in \mathbf{Z}_{t,H} \end{cases}$$

If $\tau = 0$, then $\mathbf{Z}_{t,N} = \emptyset$ and $p_t(z) = S_t p_t(z)^\star$. Trade costs create both the nontraded sector and deviations from the law of one price.

2.3 Price indices

A large number of wholesalers in each sector $i \in (F, H, N)$ bundle goods into the constant-elasticity-of-substitution (CES) composites, $C_{t,i} = \left[\left(\frac{1}{\delta_{t,i}}\right)^{\frac{1}{\theta}} \int_{\mathbf{Z}_{t,i}} c_t(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$, where $\delta_{t,i} = sup(\mathbf{Z}_{t,i}) - inf(\mathbf{Z}_{t,i})$. $\delta_{t,i}$ measures the size of the range of goods in the sector i. $c_t(z)$ is the demand for the good z and θ $(\theta > 1)$ is the intratemporal elasticity of substitution. The CES aggregation is often used in the models of monopolistic competition with differentiated products, but the aggregation here takes place across industries. The cost minimization gives the unit cost $P_{t,i} = \left[\frac{1}{\delta_{t,i}} \int_{\mathbf{Z}_{t,i}} p_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}$.

A large number of sellers combine the three baskets into final consumption in two steps. First, they bundle the export and import baskets into the CES traded basket $C_{t,T} = \left[\left(\frac{\delta_{t,H}}{\delta_{t,H}+\delta_{t,F}}\right)^{\frac{1}{\theta}} C_{t,H}^{\frac{\theta-1}{\theta}} + \left(\frac{\delta_{t,F}}{\delta_{t,H}+\delta_{t,F}}\right)^{\frac{1}{\theta}} C_{t,F}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$. This assumption is motivated by the evidence that the elasticity of substitution is greater than one (Hummels, 2001; Anderson and van Wincoop, 2004). Next, the sellers bundle the traded and nontraded baskets into the Cobb-Douglas final consumption $C_t = C_{t,T}^{s_T} C_{t,N}^{s_N} (s_N^{s_N} s_T^{s_T})^{-1}$, where s_j, $j \in (N, T)$, is the exogenous expenditure share and $s_N = 1 - s_T$. Fixed shares are assumed, to be consistent with evidence for stable shares at high frequencies in Stockman and Tesar (1995).

The cost minimization gives the traded-goods price index: $P_{t,T} = \left[\frac{\delta_{t,H}}{\delta_{t,H}+\delta_{t,F}} P_{t,H}^{1-\theta} + \frac{\delta_{t,F}}{\delta_{t,H}+\delta_{t,F}} P_{t,F}^{1-\theta}\right]^{\frac{1}{1-\theta}}$, and the following expenditure share $s_{t,i}, i \in (F, H)$:

$$s_{t,i} = \frac{\delta_{t,i}}{\delta_{t,i} + \delta_{t,-i}} \left(\frac{P_{t,i}}{P_{t,T}}\right)^{1-\theta} s_T. \tag{2}$$

The consumer price index (CPI) is the geometric average of the traded- and nontraded-goods price indices, $P_t = P_{t,T}^{s_T} P_{t,N}^{s_N}$.

2.4 Real exchange rate

Define the real exchange rate (RER) as $Q_t = S_t P_t^\star / P_t$, where S_t is the nominal exchange rate. Naknoi (2008) shows that the RER is the product of the traded and nontraded RERs, $Q_t = Q_{t,T} Q_{t,N}$, where

$$Q_{t,T} = \frac{S_t P_{t,T}^\star}{P_{t,T}} = \left[\frac{s_{t,H}}{s_T}\left(\frac{1}{1-\tau}\right)^{1-\theta} + \frac{s_{t,F}}{s_T}(1-\tau)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{3}$$

$$Q_{t,N} = \left(\frac{P_{t,N}^\star}{P_{t,T}^\star}\right)^{s_N}\left(\frac{P_{t,T}}{P_{t,N}}\right)^{s_N} \tag{4}$$

The traded RER is the relative price of the traded-goods baskets. The nontraded RER depends on international differences in the price of nontraded relative to traded goods.

Define the sector-level productivity such that the sectoral output is monotonically increasing in the product of the sectoral productivity, TFP and labor input: $A_{t,i} = \left[\frac{1}{\delta_{t,i}}\int_{\mathbf{Z}_{t,i}} a_t(z)^{\theta-1} dz\right]^{\frac{1}{\theta-1}}, i = H, N$. Since only the relative industry productivity matters, for simplicity assume that $a_t^\star(z) = 1$. Define the terms of trade as $\Omega_t = P_{t,H}/P_{t,F}$. Define the long-run home bias within the traded sector as $\bar{h} = \bar{s}_H/\bar{s}_F$. Substituting the equilibrium prices into the terms of trade, (3) and (4), gives the equilibrium path of RERs. Define $\hat{x}_t$ as the short-run deviation from the steady state of the variable x denoted by $\bar{x}$. Then,

$$\widehat{Q}_{t,T} = -\xi\widehat{\Omega}_t + \frac{\xi}{\theta-1}\widehat{Z}_t, \tag{5}$$

$$\widehat{Q}_{t,N} = s_N(\widehat{A}_{t,N} - \widehat{A}_{t,H}) - s_N\left[(1-\xi)\widehat{\Omega}_t + \frac{\xi}{\theta-1}\widehat{Z}_t\right] \tag{6}$$

where $\xi = \bar{h}[(1-\tau)^{1-\theta} - (1-\tau)^{\theta-1}]/\{(1+\bar{h})[(1-\tau)^{1-\theta} + \bar{h}(1-\tau)^{\theta-1}]\} > 0$. ξ is the elasticity of the traded RER appreciation with respect to the terms of trade. Z_t is the range of home exports relative to that of foreign exports, $z_t^l/(1-z_t^h)$.

Let var_T, var_N and $corr_{TN}$ denote the variance of the traded and nontraded RERs, and their correlation, respectively. From (5) and (6),

$$\begin{aligned} corr_{TN} \quad &= \quad \frac{s^N \xi}{\sqrt{var_T var_N}} \Bigg[(1-\xi) var(\widehat{\Omega}_t) - \frac{\xi}{(\theta-1)^2} var(\widehat{Z}_t) - \frac{1-2\xi}{\theta-1} cov(\widehat{\Omega}_t, \widehat{Z}_t) \\ &\qquad - cov(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{\Omega}_t) + \frac{1}{\theta-1} cov(\widehat{A}_{t,N} - \widehat{A}_{t,H}, \widehat{Z}_t) \Bigg]. \end{aligned} \tag{7}$$

Proposition 2 *If $\hat{z}_t^l = \hat{z}_t^h = 0$, then $|corr_{TN}| = 1$.*

Proof. If $\hat{z}_t^l = \hat{z}_t^h = 0$, then $\hat{Z}_t = \hat{A}_{t,i} = 0$. Hence, (5), (6) and (7) give $corr_{TN} = 1$ if $\xi < 1$ and $corr_{TN} = -1$ if $\xi > 1$. ■

Without the extensive margin adjustment, the traded and nontraded RERs are perfectly correlated. The perfect correlation is caused by the terms of trade, or the relative wage in this case. This result was also shown in Naknoi (2008). We display it here because it is central to the mechanism in this study.

This result has important implications for open-economy macroeconomics. The extensive margin adjustment is the mechanism that generates the weak or absence of correlation of the traded and nontraded RERs in Naknoi (2008) to match the statistics reported in Engel (1999), Mendoza (2000) and Burstein, Eichenbaum and Rebelo (2005). The weak correlation implies that the adjustment along the extensive margin reduces volatility of the RER, and hence the cost of business cycle fluctuations. These fluctuations are removed by adjustment of relative prices including the RERs or the intensive margin adjustment in the standard literature. In Proposition 1, the extensive margin adjustment is produced by volatility of the relative wage, which in turn depends on the exchange rate. We introduces sticky wages in the next section to model nominal exchange rates.

2.5 Households and sticky wages

The model features a monopolistically competitive labor market, in which the wage-setting households are indexed by $k \in (0, 1]$. The set of home residents is $[0, \alpha], \alpha \in (0, 1)$. The set of foreign residents is $(\alpha, 1]$. The home residents' optimization problem is described, and the foreign one is a mirror image. The household k's utility depends on its consumption C_t^k, its real money balance M_t^k/P_t, and its labor supply l_t^k, which depends on its wage W_t^k. Its lifetime expected utility is:

$$U_t^k = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{\sigma}{\sigma - 1} C_t^{k\,\frac{\sigma-1}{\sigma}} + \frac{\kappa_m}{1-\epsilon} \left(\frac{M_t^k}{P_t} \right)^{1-\epsilon} + \frac{\kappa_l}{\mu} (1 - l_t^k(W_t^k))^{\mu} \right].$$

$0 < \beta < 1$, $\mu < 1$, $\sigma > 0$, $\epsilon > 0$. $\kappa_m > 0$, and $\kappa_l > 0$. The household accumulates assets through money and a one-period home-currency international bond F_t^k paying the interest rate i_t. Adjusting the bond holdings is assumed to be costly, to prevent the bond holding from becoming infinitely large. Otherwise, the model cannot be solved by the log-linearization technique (Turnovsky, 1985). The cost is quadratic in deviations of the real value of bond holdings from its steady-state, assumed to be zero, $\Phi(F_t^k/P_t) = \frac{1}{2}\phi(F_t^k/P_t)^2$, $\phi > 0$.

It is also costly to adjust wages, and the cost is similar to the price adjustment cost in Rotemberg (1982). The cost is quadratic in deviations of the wage inflation from its steady state, $h(\pi_t^{w,k}) = \frac{1}{2}\phi^w \left(\pi_t^{w,k} - \bar{\pi}^{w,k} \right)^2$, $\pi_t^{w,k} = W_t^k / W_{t-1}^k$ and $\phi^w > 0$. T_t^k is the government transfer. Δ is the first difference. The budget constraint requires that the asset accumulation is the gap between income and expenditure:

$$\Delta M_t^k + \Delta F_t^k = i_t F_{t-1}^k + W_t^k l_t^k + T_t^k P_t - \left[C_t^k + \Phi(F_t^k/P_t) + h(\pi_t^{w^k}) \right] P_t.$$

The aggregate labor supply is a CES index with the substitution elasticity η, $L_t = \left[(1/\alpha)^{\eta} \int_0^{\alpha} l_t^{k(1-1/\eta)} dk \right]^{1-1/\eta}$. The unit labor cost $W_t = \left[\frac{1}{\alpha} \int_0^{\alpha} W_t^{k1-\eta} dk \right]^{\frac{1}{1-\eta}}$ is obtained from the cost minimization. The demand for the household k's labor is then $l_t^k = \frac{1}{\alpha} \left(\frac{W_t^k}{W_t} \right)^{-\eta} L_t$. The household k chooses the stochastic processes $\{W_t^k, F_t^k, M_t^k\}_{t=0}^{\infty}$ to maximize its utility subject to its budget constraint, the demand for its labor and the transversality condition $lim_{j \to \infty} E_t \left[F_{t+s}^k / \Pi_{s=0}^{j-1} (1 + i_{t+s}) \right] \geqslant 0$, taking as given the price sequence $\{P_t\}_{t=0}^{\infty}$ and the initial conditions $(W_{-1}^k, F_{-1}^k, M_{-1}^k)$. All households face an identical problem. Thus, the index k is dropped from the first-order conditions in the log-linearized form.

$$\begin{aligned}
\hat{\pi}_t^w &= \beta E_t \hat{\pi}_{t+1}^w + \lambda_w \left(\frac{1}{\sigma} \hat{C}_t + \lambda_l \hat{l}_t - \hat{w}_t \right) \\
\frac{1}{\sigma}(E_t \hat{C}_{t+1} - \hat{C}_t) &= \left(\frac{\bar{\imath}}{1+\bar{\imath}} \hat{\imath} + E_t \hat{\pi}_{t+1} \right) - \phi f_t \\
\hat{m}_t &= \frac{\epsilon}{\sigma} \hat{C}_t - \frac{\epsilon}{1+\bar{\imath}} \hat{\imath}
\end{aligned}$$

$\lambda_w = (\eta - 1)\bar{w}\bar{l}/(\beta \phi^w \bar{\pi}^2) > 0$ and $\lambda_l = \mu - 1 > 0$. w_t is the real wage W_t/P_t. Because of wage rigidity, the wage inflation rises when the marginal rate of substitution (MRS) between consumption and labor exceeds the real wage during a consumption boom. The Euler equation and the real money demand functions are typical ones in the literature.

Assume a foreign-currency bond so that the foreign interest rate $i_t^\star$ is well-defined. The bond is available only to the foreign residents and has zero supply in equilibrium. The Euler conditions yield the interest rate parity condition:

$$\hat{S}_t = E_t \hat{S}_{t+1} - \frac{\bar{\imath}}{1+\bar{\imath}}(\hat{\imath}_t - \hat{\imath}_t^\star) + \phi f_t^\star. \tag{8}$$

2.6 Degree of trade openness

The typical measure of openness is the trade-to-GDP ratio, denoted by ψ_t. It can be calculated from the optimal consumption and the budget constraint. Let b_t denote the current account relative to GDP. Then,

$$\psi_t = s_{t,F}(1 - b_t) + s_{t,H}^\star (1 - b_t^\star) \left[\frac{Y_t^\star Q_t}{Y_t(1-\tau)} \right] \tag{9}$$

With zero bond holdings in the long run, $\bar{b} = \bar{b}^\star = 0$ and the import share is the same as the export share. Hence, $\bar{\psi} = 2\bar{s}_F$. Since the expenditure share of imports and exports depend also on the extensive margin, we can decompose trade openness into the extensive and intensive margins as follows:

$$\bar{\psi} = (2\bar{z}^l)(\bar{s}_F/\bar{z}^l). \tag{10}$$

2.7 Exchange rate regimes

Exchange rate variability is formulated as an outcome of the exchange rate policy characterized by the home interest rate rule. Under the fixed exchange rate regime, the home central bank adopts the following rule.

$$\hat{\imath}_t = \hat{\imath}^\star_t + \hat{S}_t + \lambda_f f^\star_t,$$

where $\lambda_f = \phi\bar{\imath}/(1+\bar{\imath})$. $\hat{S}_t$ is the deviation from the target exchange rate and $f^\star_t = F^\star_t/(S_t P^\star_t)$. This rule and the interest parity condition in (8) give $\hat{S}_t = 0$ or fixed exchange rates. The rule is similar to that in Benigno (2004) and Monacelli (2004). Define the real gross domestic product (GDP) as $Y_t = P_t^{-1}\int_{z\in\mathbf{Z}_{t,H}\cup\mathbf{Z}_{\mathbf{t},\mathbf{N}}} p_t(z)y_t(z)dz$, and the inflation rate as π_t. The rule in Chari et al. (2002) is adopted under the flexible exchange rate regime:

$$\hat{\imath}_t = \lambda_i\hat{\imath}_{t-1} + (1-\lambda_i)\left[\lambda_\pi E_t\hat{\pi}_{t+1} + \lambda_y\hat{Y}_t\right]$$

The foreign central bank always follows a similar rule:

$$\hat{\imath}^*_t = \lambda_i\hat{\imath}^*_{t-1} + (1-\lambda_i)\left[\lambda_\pi E_t\hat{\pi}^*_{t+1} + \lambda_y\hat{Y}^*_t\right]$$

To focus on the exchange rate policy, the seigniorage revenue is assumed to be rebated to the households, $T_t = (M_t - M_{t-1})P_t^{-1}$.

2.8 Stochastic process of shocks

The focus here is on productivity shocks because the literature emphasizes the benefit from exchange rate flexibility in the presence of real shocks. The TFP shocks are characterized by the following stationary first-order autoregressive process.

$$\begin{aligned} ln(X_t) &= \rho_x ln(X_t) + u_t \\ ln(X^\star_t) &= \rho^\star_x ln(X^\star_t) + u^\star_t \end{aligned}$$

$0 < \rho_x < 1$ and $0 < \rho_x^\star < 1$. u_t and $u_t^\star$ follow the normal distribution $N(0, \sigma_u^2)$ and $N(0, \sigma_u^{\star 2})$, respectively.

2.9 Measure of welfare

The welfare loss from fixing the exchange rate is measured by the permanent decrease in consumption that the household is willing to give up to achieve the same expected utility as the floating regime. The measure is similar to that in Sutherland (2006), Devereux and Engel (2003) and Kollmann (2004). Denote the welfare loss as Γ, the expected utility under the flexible regime as V^{flex}, and the expected utility under the fixed regime as V^{fix}. The superscripts $flex$ and fix correspond to the flexible and fixed regimes, respectively. By definition,

$$V^{fix} = \sum_{t=0}^{\infty} \beta^t U((1-\Gamma)C_t^{flex}, \frac{M_t^{flex}}{P_t^{flex}}, l_t^{flex}),$$

$$V^{flex} = \sum_{t=0}^{\infty} \beta^t U(C_t^{flex}, \frac{M_t^{flex}}{P_t^{flex}}, l_t^{flex}).$$

Differencing the two values of the expected utility and rearranging terms give the solution for Γ:

$$\Gamma = 1 - \left(1 + \frac{V^{flex} - V^{fix}}{\sum_{t=0}^{\infty} \beta^t U(C_t^{flex}, 0, 1)}\right)^{\frac{\sigma}{\sigma-1}}. \tag{11}$$

The quantitative predictions of the model are in the next section.

3 Calibration

Table 1 tabulates the benchmark parameters. The parameters related to specialization are selected so that the elasticity of the relative set of exports with respect to relative country size does not exceed the estimate in Hummels and Klenow (2005). The trade cost parameter is from Hummels (2001). Most parameters follow the standard business cycle literature such as Mendoza (1991) and Chari et al. (2002). The interest rate rule under the flexible exchange rate regime is the estimated rule from the U.S. data by Clarida, Gali and Gertler (2000). With symmetric country size, the degree of trade openness is 0.21. This is slightly higher than openness of the U.S. but comparable to that of the U.K.

3.1 Benchmark calibration results

Table 2 summarizes the results from 50 simulations. Reported statistics are the average of all simulations, except for the welfare loss from fixing exchange rates. Column 1 corresponds to the model in this study. However, to understand the effect of extensive margin adjustment we need to compare the statistics with comparable ones obtained from the model with only adjustments along the intensive margin. For this reason, Column 2 reports the corresponding statistics from an alternative model in which the pattern of specialization is given by the steady state of the benchmark model. The last column presents the difference in statistics from the model with endogenous specialization and the model with exogenous specialization. In other words, the last column reports the effects of extensive margin adjustments.

There are five main findings. First, with endogenous specialization the welfare loss is positive for both countries. Hence, the flexible exchange rate regime dominates the fixed exchange rate regime. Given that the monetary rule for the flexible exchange rate regime is not an optimized rule which can yield higher welfare, this result implies that the flexible exchange rate regime is desirable. This prediction is in line with Devereux and Engel (2003), given the absence of local currency pricing.

Second, the extensive margin adjustment increases the welfare loss by roughly 1 percent in the home country and half a percent in the foreign country. The magnitude is large compared to the welfare loss generated by the intensive margin in Column 2.

Third, the extensive margin reduces volatility of the RER and the terms of trade, but raises volatility of the current account-to-GDP ratio and the trade-balance-to-GDP ratio. The reason is that positive TFP shocks drive up the relative wage and cause low-productivity exporters to quit exporting. The exit of low-productivity firms increases the average productivity in the export sector, which offsets the impact of rising relative wage on the terms of trade. The fall of the terms of trade volatility then reduces volatility of the RER. The resulting decrease of volatility of the quantity margin of trade is intuitive since now there is an additional adjustment margin.

Fourth, the offsetting effect of average productivity fluctuations on terms of trade is so large that the correlation between the traded and nontraded RERs is virtually zero. This result confirms the prediction in Proposition 2. Moreover, Engel (1999) found that the correlation in the U.S. RER data is zero. This result has been documented by Naknoi (2008), and we report it here as a consistency check.

Finally, adjustments along the extensive margin reduce the correlation between relative consumption of the two economies and the RER, from roughly 0.8 to 0.55. Although our correlation is much higher than that in Backus and Smith (1993), it is lower than 0.71 in Ghironi and Melitz (2005).

To summarize, the quantitative results suggest that adjustments along the extensive margin significantly increase the dynamic adjustments through trade and current account. Hence, adjustments along the extensive margin reduce variability of the RER and increase the benefit from exchange rate flexibility.

Next, we vary the parameters for country size and trade costs to study the impact of trade openness on the benefit from exchange rate flexibility.

3.2 Effects of country size

Figure 1 plots the degree of trade openness against the size of the home country. The degree of openness is decreasing in country size. When we decompose openness according to (10), the extensive margin of openness is increasing in country size, but the intensive margin of openness is decreasing in country size. This finding is consistent with the finding in Hummels and Klenow (2005). In our model, the large country exports a wide range of goods because its large labor supply drives down the relative wage. Large labor supply also yields large output for consumption and thus reduces the expenditure share of imports.

Figure 2 plots the welfare loss from fixing exchange rate against the size of the home country. The welfare loss is non-monotonic in country size. However, the loss along the extensive margin is increasing in country size, but the loss along the intensive margin is non-monotonic in country size. The non-monotonicity arises from the fact that adjustments along the intensive margin come from both price and quantity adjustments. Since demand falls when price rises, the effect of price changes on the value of consumption is ambiguous. Consequently, the intensive margin drives the non-monotonicity between the welfare loss and degree of trade openness.

3.3 Effects of trade costs

Figure 3 plots the degree of trade openness against trade costs. The degree of openness is decreasing in trade costs, and so is the extensive margin of openness. This is because high trade costs expands the range of nontraded goods. However, rising trade costs increase the intensive margin of openness. When trade costs rise, prices of traded goods rise as a result. Hence, our results imply that the reduction of demand is small, and therefore the expenditure on each traded good is increasing in trade costs.

Figure 4 plots the welfare loss from fixing exchange rate against trade costs. The welfare loss is non-monotonic in trade costs. The non-monotonicity exists when we decompose the loss into the extensive and intensive margins too. The non-monotonicity of the intensive margin arises from the same mechanism as when we vary the country size. However, the non-monotonicity of the welfare loss along the extensive margin is particularly interesting.

When trade costs rise, the set of home exports and the set of home imports contract at the same time, since trade costs determine not only the relative wage but also the relative productivity schedule faced by the two countries. As a result, the number of firms switching in and out of exporting will be small unless exchange rate fluctuations are large.

In contrast, a rise in country size influences only the real wage, keeping the relative productivity schedule unchanged. Hence, it has the opposite effect on the set of home exports and the set of home imports. In fact, the width of the range of nontraded goods is not influenced by country size, although a change in country size shifts the range of nontraded goods. For this reason, an increase in country size does not diminish the extensive margin adjustment.

4 Concluding remarks

This study investigates the effects of trade openness on the benefit from exchange rate flexibility. We find no evidence that fixing exchange rates is desirable. Although the monetary rule under the flexible exchange rate regime in this study is not an optimized rule, it is sufficient to say that flexible exchange rate policy dominates fixed exchange rate policy. This is because by construction an optimal policy will increase the welfare under the flexible regime, and therefore it will make the flexible regime even more desirable.

Our work lends support to the argument that a small open economy does not need to include the exchange rate in the monetary rule in Clarida et al. (2002). It produces the same prediction as Adrian and Gros (2004), although their arguments are based on a different mechanism. In Adrian and Gros (2004), a high degree of trade openness can increase the cost of fixing exchange rates when foreign shocks are large. However, our result holds despite the symmetry of size of shocks in the two countries.

When we decompose the benefit of exchange rate flexibility into the extensive and intensive margins. We find that openness has different impacts on the benefit along the extensive and intensive margins. To be specific, the benefit is increasing in the degree of openness only when it is measured as the benefit along the extensive margin *and* variations in trade openness is driven by variations in country size. As demonstrated by the quantitative exercise, the benefit along the extensive margin is no more monotonic in the degree of openness when variations in trade openness are driven by trade costs.

However, the benefit along the intensive margin is always non-monotonic in the degree of trade openness. For this reason, the combined benefit along the two adjustment margins is non-monotonic in the degree of trade openness.

Apart from the welfare comparison, our model can also generate two important business cycle properties of the RER. The model yields zero correlation between the traded and nontraded RERs as documented by Engel (1999). It also reduces the correlation between the RER and relative consumption. The predicted correlation is closer to the estimated zero correlation in Backus and Smith (1993) than the study by Ghironi and Melitz (2005). Hence, our model has the potential to explain the lack of correlation between the RER and relative consumption. Taking into account the firm's investment decisions is a natural extension of this study.

References

Adrian, T. and Gros, D. (2004). The degree of openness and the cost of fixing exchange rate. *Economics Letters*, 83(1):141–146.

Anderson, J. E. and van Wincoop, E. (2004). Trade costs. *Journal of Economic Literature*, 42(3):691–751.

Backus, D. K. and Smith, G. W. (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, 35(3-4):297–316.

Benigno, G. (2004). Real exchange rate persistence and monetary policy rules. *Journal of Monetary Economics*, 51(3):473–502.

Bergin, P. R. and Corsetti, G. (2002). The extensive margin and monetary policy. *Journal of Monetary Economics, forthcoming*.

Besedes, T. and Prusa, T. J. (2006). Ins, outs and the duration of trade. *Canadian Journal of Economics*, 39(1):266–295.

Burstein, A., Eichenbaum, M., and Rebelo, S. (2005). Large devaluations and the real exchange rate. *Journal of Political Economy*, 113(4):742–784.

Castellanos, S. G., Garcia-Verdu, R., and Kaplan, D. S. (2004). Nominal wage rigidities in mexico: Evidence from social security records. *Journal of Development Economics*, 75(2):507–533.

Chari, V. V., Kehoe, P. J., and McGrattan, E. R. (2002). Can sticky price models generate volatile and persistent real exchange rates? *The Review of Economic Studies*, 69:533–563.

Clarida, R., Gali, J., and Gertler, M. (2000). Monetary policy rules and macroeconomic stability: Evidence and some theory. *Quarterly Journal of Economics*, 115:147–180.

Clarida, R., Gali, J., and Gertler, M. (2002). A simple framework for international monetary policy analysis. *Journal of Monetary Economics*, 49:879–904.

Devereux, M. B. and Engel, C. (2003). Monetary policy in the open economy revisited: Price setting and exchange-rate flexibility. *Review of Economic Studies*, 70(4):765–783.

Dornbusch, R., Fischer, S., and Samuelson, P. A. (1977). Comparative advantage, trade, and payments in a Ricardian model with a continuum of goods. *American Economic Review*, 67(5):823–839.

Engel, C. (1999). Accounting for U.S. real exchange rate changes. *Journal of Political Economy*, 107(3):507–538.

Falvey, R. E. and Gemmell, N. (1995). Explaining international differences in the share of services in real expenditure. *Economic Letters*, 47:53–58.

Frenkel, J. A. and Aizenman, J. (1982). Aspects of the optimal management of exchange rates. *Journal of International Economics*, 13(3-4):231–256.

Friedman, M. (1953). The case for flexible exchange rates. In *Essays in Positive Economics*, pages 157–203. Chicago University Press.

Ghironi, F. and Melitz, M. J. (2005). International trade and macroeconomic dynamics with heterogeneous firms. *Quarterly Journal of Economics*, 120(3):865–915.

Huang, K. X. and Liu, Z. (2002). Staggered price-setting, staggered wage-setting and business cycle persistence. *Journal of Monetary Economics*, 49:405–433.

Hummels, D. (2001). Toward a geography of trade costs. Purdue University, Working Paper.

Hummels, D. and Klenow, P. (2005). The variety and quality of a nation's trade. *American Economic Review*, 95(3):704–723.

Kahn, S. (1997). Evidence of nominal wage stickiness from microdata. *American Economic Review*, 87(5):993–1008.

Kollmann, R. (2004). Welfare effects of a monetary union: the role of trade openness. *Journal of European Economic Association*, 2(2-3):289–301.

Liu, Z. and Phaneuf, L. (2007). Technology shocks and labor market dynamics: Some evidence and theory. *Journal of Monetary Economics*, 54(8):2534–2553.

Mendoza, E. G. (1991). Real business cycles in a small open economy. *American Economic Review*, 81(4):797–818.

Mendoza, E. G. (2000). On the stability of variance decompositions of the real exchange rate across exchange-rate regimes: Evidence from Mexico and the United States. *NBER Working Papers*, 7768.

Monacelli, T. (2004). Into the Mussa puzzle: Monetary policy regimes and the real exchange rate in a small open economy. *Journal of International Economics*, 62(1):191–217.

Naknoi, K. (2008). Real exchange rate fluctuations, endogenous tradability and exchange rate regimes. *Journal of Monetary Economics*, 55(3):645–663.

Rotemberg, J. J. (1982). Sticky prices in the united states. *Journal of Political Economy*, 90:1187–1211.

Sutherland, A. (2006). The expenditure switching effect, welfare and monetary policy in a small open economy. *Journal of Economic Dynamics and Control*, 30(7):1159–1182.

Turnovsky, S. J. (1985). Domestic and foreign disturbances in an optimizing model of exchange rate determination. *Journal of International Money and Finance*, 4(1):151–171.

Table 1: Benchmark parameter values

Parameters	Value
Specialization	
(*Hummels and Klenow, 2005; Hummels, 2001*)	
Country size	$\alpha = 0.5$
Relative productivity	$n = 1.5, \gamma = 1$
Entry-cost parameter	$\phi_a = 9$
Trade costs	$\tau = 0.15$
Households	
Intratemporal elasticity of substitution	$\theta = 3$
(*Anderson and van Wincoop, 2004*)	
Intertemporal elasticity of substitution	$\sigma = 0.2$
(*Chari et al., 2002*)	
Expenditure share of nontraded goods	$s_N = 0.5$
(*Falvey and Gemmell, 1995*)	
Discount factor	$\beta = 0.99$
(*Chari et al., 2002*)	
Elasticity of labor supply	$\mu = 1 - 1/\sigma$
(*Chari et al., 2002*)	
Interest semi-elasticity of money demand	$1/\epsilon = 0.39$
(*Chari et al., 2002*)	
Portfolio adjustment cost	$\phi = 0.00074$
(*Mendoza, 1991*)	
Labor market	
(*Huang and Liu, 2002*)	
Elasticity of substitution of labor	$\eta = 4$
Wage adjustment cost	$\phi^w = 5.8935$
Monetary policy	
(*Clarida, Gali and Gertler, 2000*)	
Steady-state inflation	$\bar{\pi} = \bar{\pi}^\star = 1.0358^{0.25}$
Interest rate rule	$\lambda_i = 0.79$, $\lambda_\pi = 2.15$, $\lambda_y = 0.93$
Productivity shock	
(*Chari et al., 2002*)	
Persistence	$\rho_x = 0.95$
Volatility	$\sigma_u = \sigma_u^\star = 0.01$, $\sigma_{u,u^\star} = (0.25)0.01^2$

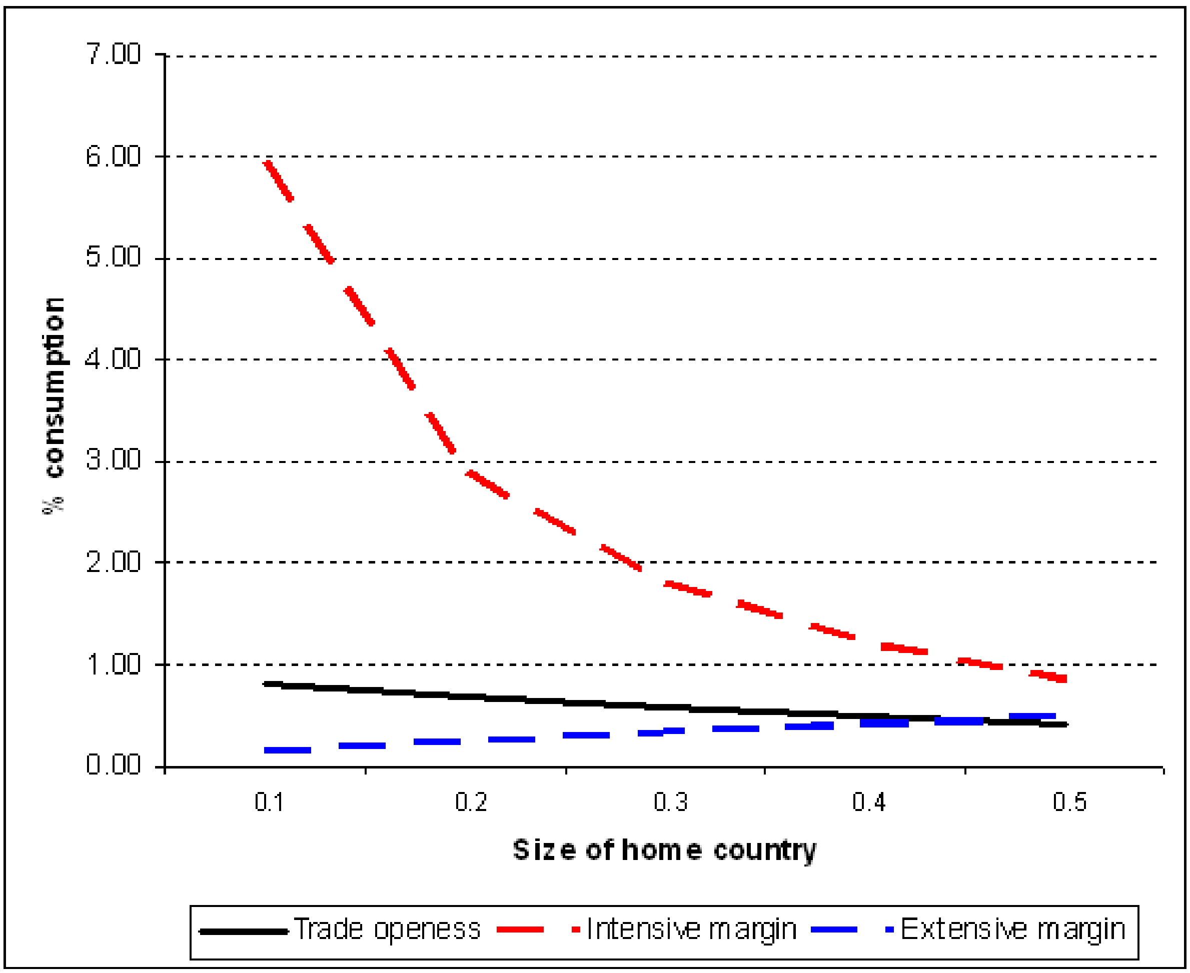
Table 2: Benchmark calibration results

	Endogenous specialization	**Exogenous specialization**	**Extensive margin effect**
Welfare loss			
Home country	0.013	0.004	0.009
Foreign country	0.005	-0.0004	0.006
Standard deviation			
Real exchange rate			
Fixed regime	2.03	4.03	-1.99
Flexible regime	2.29	4.30	-2.01
Terms of trade			
Fixed regime	2.23	4.34	-2.11
Flexible regime	2.51	4.66	-2.15
Current account/GDP			
Fixed regime	5.08	4.10	0.98
Flexible regime	5.92	5.10	0.81
Trade balance/GDP			
Fixed regime	2.16	1.94	0.22
Flexible regime	2.41	2.04	0.38
Correlation			
Traded and nontraded real exchange rates			
Fixed regime	0.05	1	-0.95
Flexible regime	0.07	1	-0.93
Data (Engel, 1999)	0	0	
Relative consumption and real exchange rate			
Fixed regime	0.55	0.81	-0.26
Flexible regime	0.54	0.80	-0.26
Data (Backus and Smith, 1993)	0.05	0.05	

Notes:

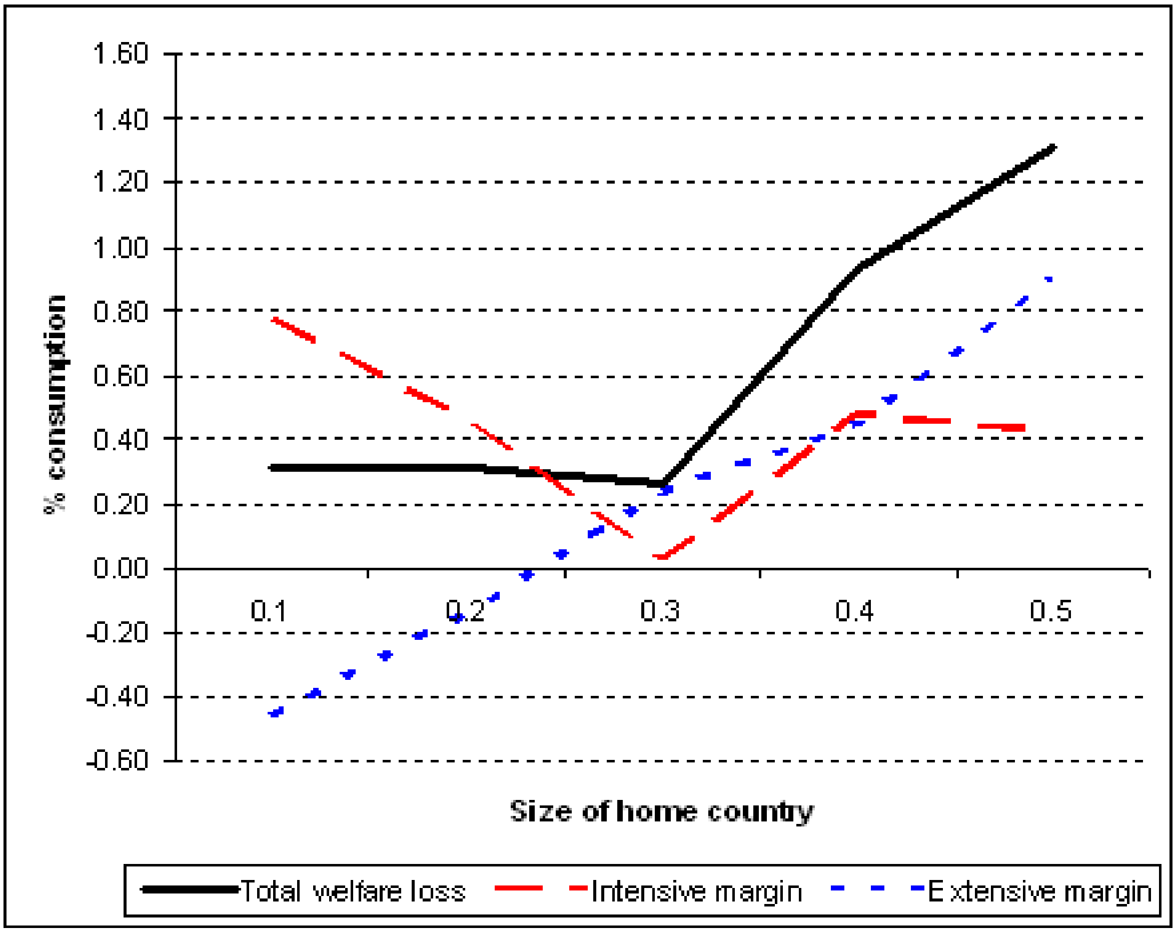
1. Welfare loss is measured by percentage decrease of flexible-regime consumption that yields the same expected lifetime utility as the allocation of consumption, real money balance and labor under the fixed exchange rate regime.
2. Except for the welfare loss, reported statistics are the average over 50 simulations.
3. Standard deviations are measured relative to the standard deviation of output.

Figure 1: Country size and degree of trade openness



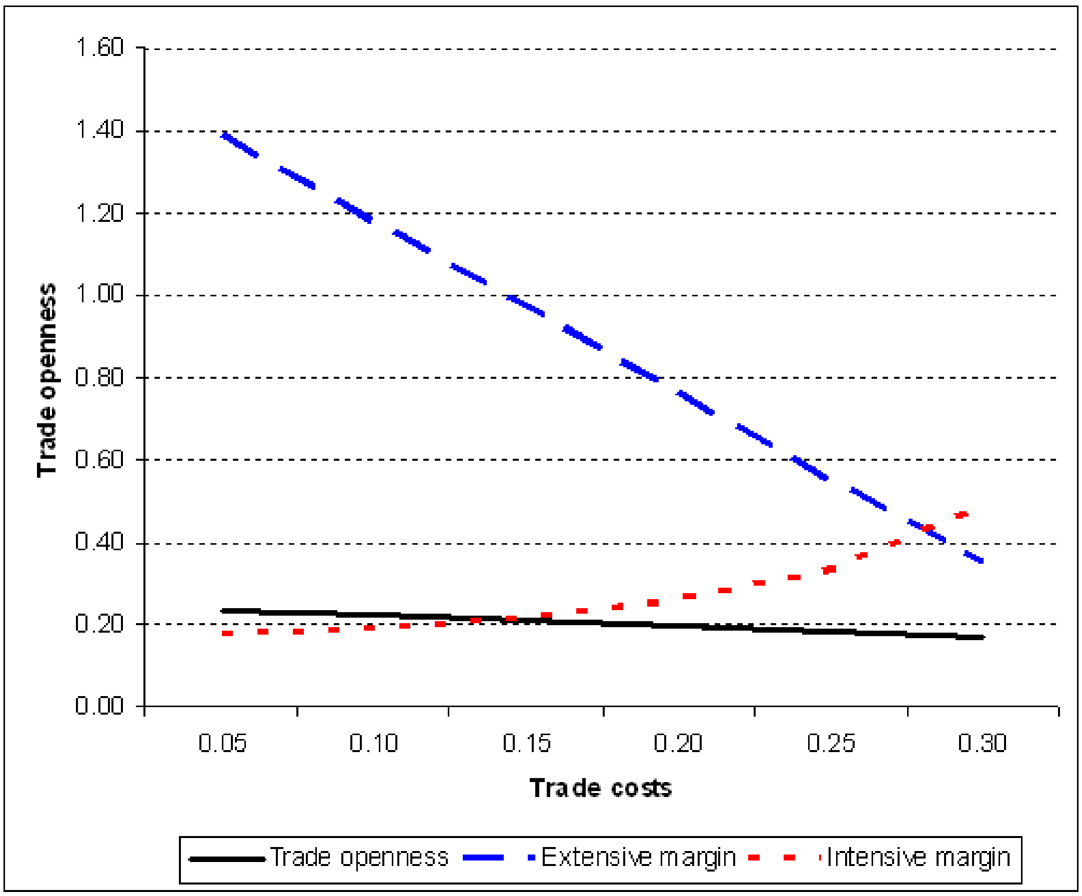
Note: Trade openness = (value of exports + value of imports)/GDP

Figure 2: Country size and welfare loss from fixing exchange rates



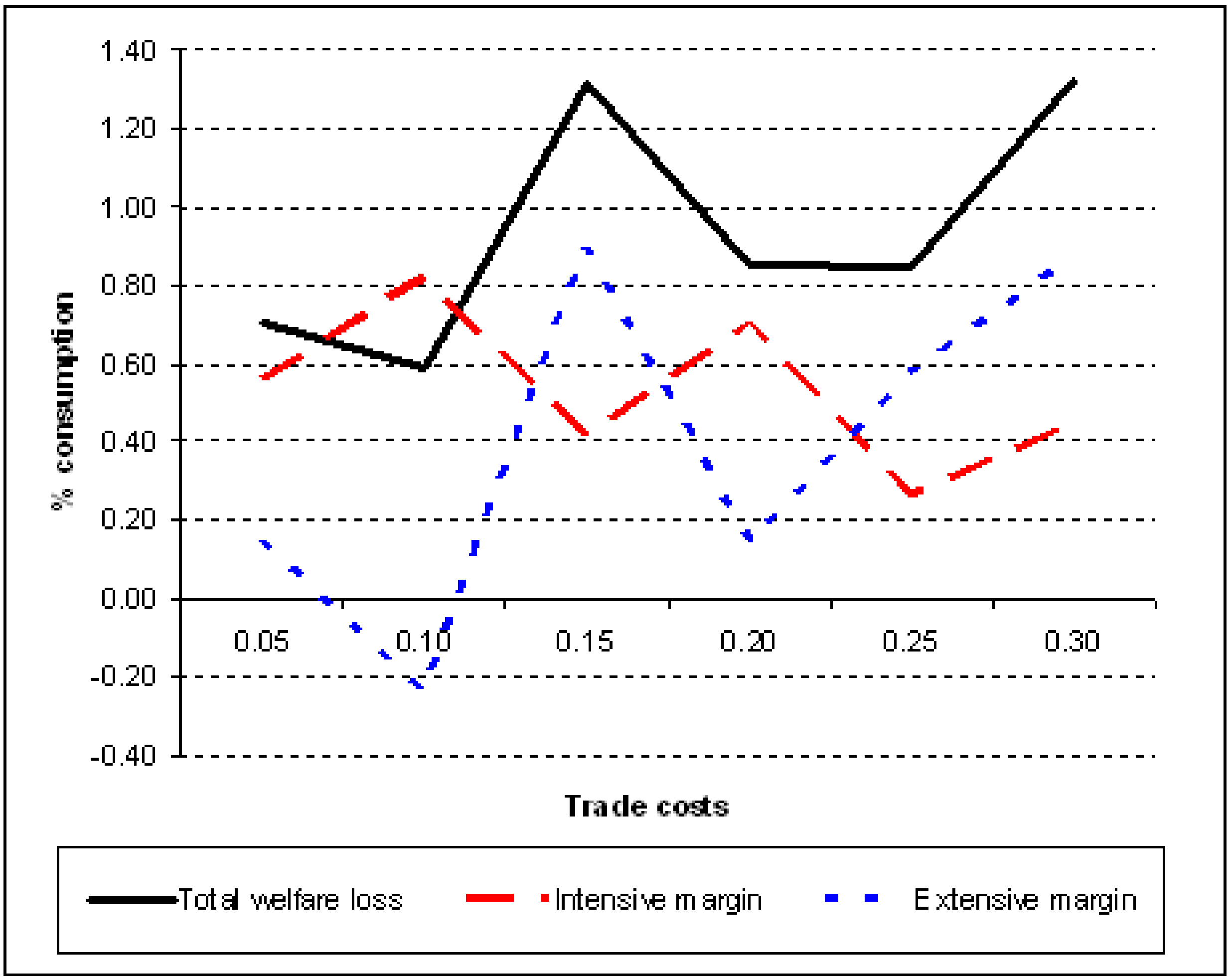
Notes:
1. Welfare loss is measured by percentage decrease of flexible-regime consumption that yields the same expected lifetime utility as the allocation of consumption, real money balance and labor under the fixed exchange rate regime.
2. Total welfare loss = welfare loss along the extensive margin + welfare loss along the intensive margin.

Figure 3: Trade costs and degree of trade openness



Note: Trade openness = (value of exports + value of imports)/GDP

Figure 4: Trade costs and welfare loss from fixing exchange rates



Notes:
1. Welfare loss is measured by percentage decrease of flexible-regime consumption that yields the same expected lifetime utility as the allocation of consumption, real money balance and labor under the fixed exchange rate regime.
2. Total welfare loss = welfare loss along the extensive margin + welfare loss along the intensive margin.