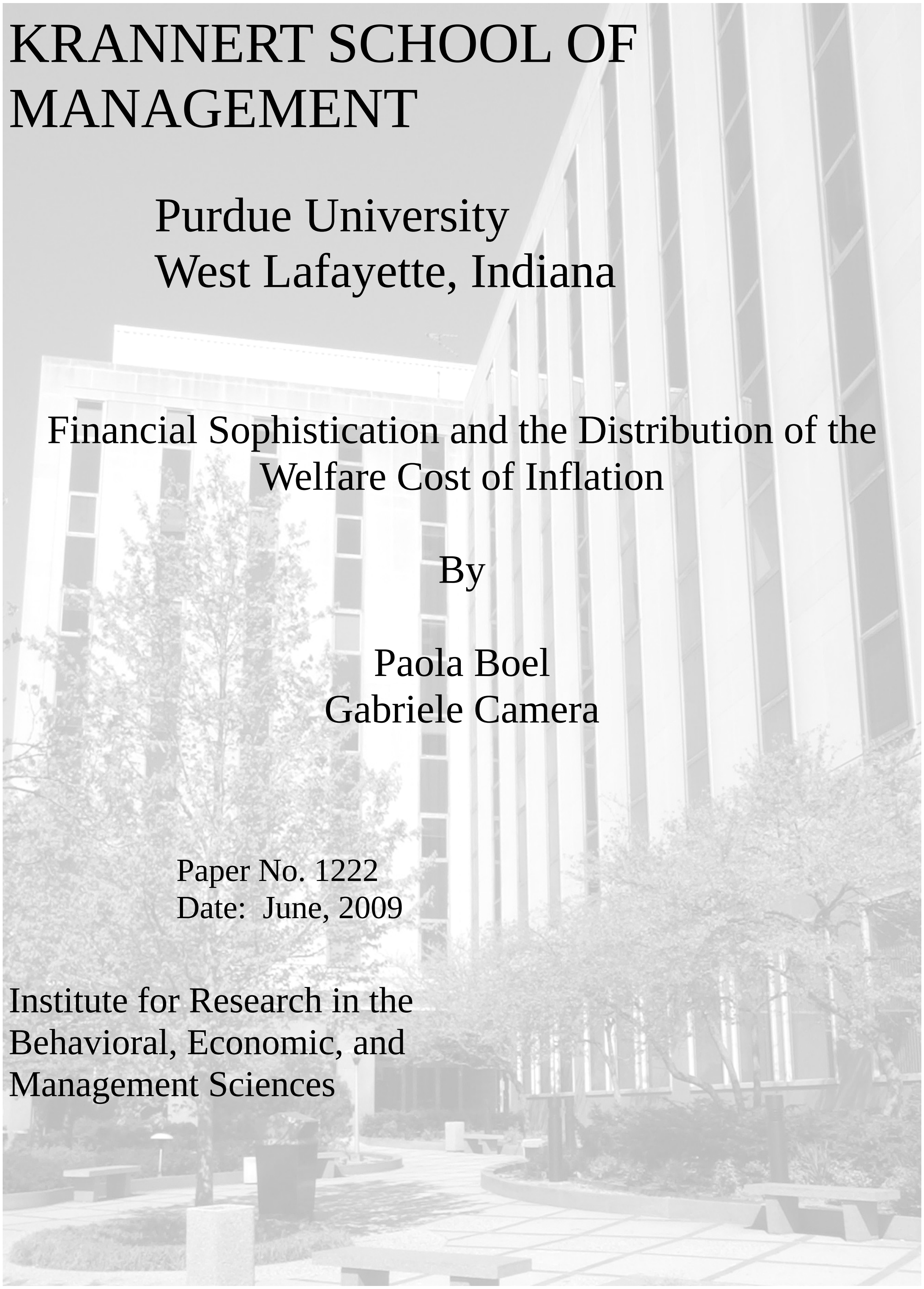

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Financial Sophistication and the Distribution of the Welfare Cost of Inflation

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Paper No. 1222
Date: June, 2009

Institute for Research in the Behavioral, Economic, and Management Sciences

Financial Sophistication and the Distribution of the Welfare Cost of Inflation[†]

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Abstract

The welfare cost of anticipated inflation is quantified in a calibrated model of the U.S. economy that exhibits tractable equilibrium dispersion in wealth and earnings. Inflation does not generate large losses in societal welfare, yet its impact varies noticeably across segments of society depending also on the financial sophistication of the economy. If money is the only asset, then inflation hurts mostly the wealthier and more productive agents, while those poorer and less productive may even benefit from inflation. The converse holds in a more sophisticated financial environment where agents can insure against consumption risk with assets other than money.

Keywords: Money, Heterogeneity, Friedman rule, Trade frictions, Calibration. JEL codes: E4, E5

[†]Helpful suggestions from an anonymous Referee and the Editor are acknowledged. The

authors thank Julián P. Díaz, Paul Hanouna, Randall Wright and seminar participants at the Cleveland Fed, Midwest Macroeconomic Meetings, and SAET 2007 meetings for comments on earlier versions of this paper. G. Camera acknowledges research support from the NSF grant DMS-0437210 and thanks the people of the Institute for Advanced Studies (ISA) at the University of Bologna for their hospitality during the Spring 2009.

1 Introduction

A considerable amount of theoretical work, based on disparate modeling approaches, supports the notion that efficiency in a monetary economy is inconsistent with inflationary policy. Yet, low predictable inflation is widely tolerated and sometimes advocated. This discrepancy gives special relevance to a literature aimed at quantifying the social cost of inflation and its distributional impact. A first strand of recent studies is based on models where trade frictions provide explicit micro foundations for money. These studies usually assume money is the only store of value and if they admit heterogeneity, then they must do some heavy lifting to compute analytically complex monetary distributions. A second strand includes works based on models that often exhibit heterogeneity or a more sophisticated financial environment in which, however, money has a more "descriptive" role.[1] The present work ties together these strands of literature.

Tractable forms of ex-ante heterogeneity are introduced in a matching model of money where money has an explicit medium of exchange function and there is no role for private credit. The model is based on Lagos and Wright (2005), Boel and Camera (2006) and Aliprantis, Camera and Puzzello (2007). Equilibrium exhibits a tractable form of heterogeneity in wealth and earnings that allows an assessment, analytical and quantitative, of the distributional impact of inflation. The model is calibrated to the

[1]The first strand includes matching models as in Aruoba, Waller, and Wright (2007), Chiu and Molico (2007a,b), Lagos and Wright (2005), Molico (2006), or Reed and Waller (2006). The second strand includes precautionary balances models, cash-in-advance with costly credit, or store-of-value models as in Akyol (2004), Erosa and Ventura (2002), or Imrohoroglu (1992).

U.S. economy and it is found that the welfare cost of inflation is small on average but it is unequally distributed depending on heterogeneity and financial sophistication. In the typical setting of a financially unsophisticated economy (money is the only asset) inflation is a burden mostly or only for the wealthier and more productive segment of society, and can even be advantageous for those poorer and less productive. However, the distributional impact of inflation may change with greater financial sophistication, i.e., when agents can insure against consumption risk by means other than money.

In the benchmark model agents can hold only money to insure against consumption risk, as in the typical model of this class. In a calibrated representative-agent version of this model, ten percent inflation is worth around one percent of consumption, which is in line with previous studies; e.g., Cooley and Hansen (1989), Lucas (2000), Lagos and Wright (2005) to name a few. Subsequently, heterogeneity is introduced in labor productivity or in trade shocks, considering two types of agents for analytical tractability. The calibrated model still generates a low average welfare cost of inflation, but inflation's burden is now unequally distributed in society. Heterogeneity in trade risk supports equilibrium dispersion in monetary wealth as those who are more likely to trade save more than average; wealth inequality vanishes as nominal interest rates approach zero. Heterogeneous productivity supports dispersion in earnings but not in money holdings, because the structure of the model eliminates wealth effects.

With heterogeneity, wealthier and more productive agents suffer the most from inflation, while poorer and less productive agents suffer less and can in fact benefit from it, i.e., they would require compensation to *avoid* inflation. The reason is that inflation greatly penalizes average earnings of the more productive and, with equilibrium dispersion in money balances, it also creates unequal inflation-tax burdens that redistribute monetary wealth top-to-bottom. On the one hand, these redistributive implications are in line with quantitative and theoretical findings in models of the same class (e.g., Berentsen, Camera and Waller 2005, Chiu and Molico 2007a, Molico 2006). On the other hand, they are at odds with the empirical observation that richer agents tend to be less concerned about inflation than the poor (e.g., see Albanesi, 2007) and also with the distributional results in Erosa and Ventura (2002).

To investigate these disparities, the economy's financial sophistication is augmented by introducing a nominal asset in addition to money. This asset is traded on a prototypical financial market, can provide consumption insurance, much as money, but it can better shield agents from the inflation tax. The augmented model retains heterogeneous trade shocks and assumes finance generates no resource costs. It is shown that at small to moderate inflation rates an outcome exists in which only agents who trade and consume less than average choose to hold money, while the rest only hold the asset. Inflation in this case has still a negative impact on societal welfare. However, the impact is quantitatively smaller than before and the redistributive effects of inflation are reversed. Now it is the poor who would give up consumption to avoid inflation, while the wealthy would demand *more* consumption. The reason is that only poor agents are now subject to the inflation tax. The basic lesson from this simple model is that the assumed financial structure not only can affect the welfare cost that inflation imposes on society as a whole, but it can also have a significant impact on how the burden of inflation is distributed across society.

The paper proceeds as follows. Section 2 presents the model. Section 3 studies stationary monetary equilibrium. Section 4 discusses the calibration procedure and reports the quantitative findings for an economy with only money, while Section 5 discusses the case of a financially more sophisticated economy. Section 6 concludes.

2 The model

Time is discrete, the horizon is infinite and there is a large population of heterogeneous infinitely-lived agents who consume perishable goods and discount only even to odd dates. So, consider trading cycles indexed by $t = 1, 2, ...$ each with an odd and an even date. As in Boel and Camera (2006) there are infinitely many spatially separated trade groups each defining a market with infinitely many anonymous agents who have not met before. Thus, in each trading cycle agents may visit two anonymous markets, denoted 'one' and 'two' on odd and even dates.

On every date a single perishable consumption good can be supplied by producers, i.e., agents who can transforms each unit of their labor into one good. Everyone can produce and consume on even dates. Instead, at the start of each odd date agents draw i.i.d. trade shocks determining whether the agent can trade on market one, i.e., can either produce, consume, or do neither (idle). Consuming or producing are equally likely. Hence, on odd dates agents face idiosyncratic trade (consumption) risk, but not on even dates. Ex-ante heterogeneity is also introduced, in one of two forms; agents can either differ in their odd-date trade shocks, or productivity.[2] For convenience the population is divided into two types $j = H, L$ in proportions ρ and $1-\rho$.

Even-date preferences are assumed homogeneous and quasilinear. An agent of type j who consumes $q_j \geq 0$ goods and supplies $x_j \geq 0$ labor in market two (equivalently, produces x_j goods) has utility $U(q_j) - x_j$. On odd dates consumers of any type j derive utility $u(c_j)$ from $c_j \geq 0$ consumption. Producers of type j suffer $\phi_j(y)$ disutility from producing y goods. The functions u, ϕ_j and U are twice continuously differentiable, strictly increasing, with $u'' < 0$, $\phi_j' > 0$ and $U'' < 0 < \phi''$. Also, $\phi_j(0) = 0$ and denote with a star the quantities that uniquely solve $u'(c) = \phi_j'(c)$ and $U'(q) = 1$. There is heterogeneity in trade shocks when α_j is the probability of trading on market one for a type j, with $0 < \alpha_L < \alpha_H \leq 1$. There is heterogeneity in productivity if $\phi_H'(y) < \phi_L'(y)$ for each $y > 0$. Agents are price takers and trade under limited enforcement and limited commitment, which given the frictions considered implies an essential role for money (Aliprantis, Camera and Puzzello, 2007). A government exists that is the sole supplier of fiat currency, of which there is an initial stock $\bar{M} > 0$ evolving deterministically at

[2] See Bhattacharya, Haslag and Martin (2005) or Andolfatto (2009) for period-utility heterogeneity in a similar model.

gross rate π thanks to lump-sum transfers in market two.

3 Stationary monetary allocations

Consider the allocation selected by a planner who maximizes the agents' lifetime utilities, treating them identically, and constrained by the same physical and informational restrictions faced by agents. Such allocation, called the efficient allocation, is unique and stationary across trade cycles.[3] The planner equates the marginal rates of substitution of the different types of agents, on each date. Hence, in what follows the analysis focuses on stationary monetary outcomes. These are outcomes in which consumption is invariant across trade cycles and the sequence of nominal prices evolves so that the money stock has constant positive real value.

For simplicity omit t subscripts and use a prime to identify next-cycle variables. Accordingly, p_1 and p_2 denote the nominal price of goods on odd/even dates (markets one/two) of an arbitrary trade cycle t. Also, normalize nominal variables by p_2, so in market one the real price is $p = \frac{p_1}{p_2}$. The timing of events during cycle t for the arbitrary agent of type j is as follows. He enters cycle t with real money holdings $m_j \geq 0$, saved in the preceding cycle. After market one closes the agent enters market two on the even date with $m_{j,k}$ real balances, where $k = n, s, b$ denotes the idiosyncratic trade shock experienced in market one (n if idle, b for buyer, s for producer).

[3] This is the same allocation that would arise if agents could coordinate and commit to a non-monetary trading plan on each odd date, before realizing their individual shocks.

Individual (real) balances evolve within the cycle according to

$$m_{j,b} = m_j - pc_j, \quad m_{j,s} = m_j + py_j, \quad \text{and} \quad m_{j,n} = m_j. \tag{1}$$

In market one, a buyer spends pc_j and a producer earns py_j. In market two, the real price is one, q_j is consumption, $x_{j,k}$ is production of an agent who received shock k, and agents save $m'_j \geq 0$ real balances to self-insure against future consumption shocks. Short selling is not allowed and agents cannot lend to each other.

In a stationary monetary economy $m'_j = m_j > 0$. So, if M is the nominal money supply at the start of a cycle and $M' = \pi M$ is money available in market two, then $\frac{p'_2}{p_2} = \frac{M'}{M} = \pi$. The money growth rate (i.e., the inflation rate) is controlled via per-capita lump-sum transfers τ in market two. If every type j holds the same amount of money m_j, then the government budget constraint is

$$\tau = [\rho m_H + (1-\rho)m_L](\pi - 1). \tag{2}$$

Given money market clearing, the stationary real money stock $\bar{m} = \frac{M}{p_2}$ is

$$\bar{m} = \rho m_H + (1-\rho)m_L. \tag{3}$$

Given the recursive nature of the problem, a dynamic programming approach is used to describe the problem faced by an agent of type j on any date. Let $V_j(m_j)$ be the agent's expected lifetime utility when he starts a trade cycle with $m_j > 0$ balances before trade shocks are realized. Let $W_j(m_{j,k})$ be the expected lifetime utility from entering an even date with $m_{j,k} \geq 0$ balances.

The agent's budget constraint at the start of an even date is

$$x_{j,k} = q_j + \pi m_j' - (m_{j,k} + \tau), \tag{4}$$

where available resources partly depend on the realization of the shock k. Hence,

$$W_j(m_{j,k}) \quad = \max_{q_j, m_j' \geq 0} \{U(q_j) - q_j - \pi m_j' + m_{j,k} + \tau + \beta V_j(m_j')\}, \tag{5}$$

so $W_j(m_{j,k}) = W_j(0) + m_{j,k}$ and the marginal valuation of money is type-independent, $\frac{\partial W_j(\omega_{j,k})}{\partial m_{j,k}} = 1$ for all j. The savings choice m_j' is independent of trading histories but may be type-dependent. However, everyone consumes identically in market two since (5) implies $q_j = q^*$ for all j, so

$$W_j(m_{j,k}) = U(q^*) - q^* + m_{j,k} + \tau + \max_{m_j' \geq 0} [-\pi m_j' + \beta V_j(m_j')]. \tag{6}$$

Goods market clearing implies

$$q^* = (1-\rho)[\tfrac{\alpha_L(x_{L,s}+x_{L,b})}{2} + (1-\alpha_L)x_{L,n}] + \rho[\tfrac{\alpha_H(x_{H,s}+x_{H,b})}{2} + (1-\alpha_H)x_{H,n}]. \tag{7}$$

In a monetary economy $m_j' > 0$, hence the first order condition is $1 = \frac{\beta}{\pi} \times \frac{\partial V_j(m_j')}{\partial m_j'}$. Savings m_j' depend on the expected marginal benefit of holding money in market one, $\frac{\partial V_j(m_j')}{\partial m_j'}$, which may differ across types j, as shown next.

For a type j holding m_j balances at the start of market one

$$V_j(m_j) = \max \tfrac{\alpha_j}{2}[u(c_j) + W_j(m_{j,b}) - \phi_j(y_j) + W_j(m_{j,s})] + (1-\alpha_j)W_j(m_{j,n}) \tag{8}$$

where the maximization is over $c_j \leq \frac{m_j}{p}$ as a buyer and $y_j \geq 0$ as a producer. If $y_j > 0$ for all j, then optimality in market one requires

$$p = \phi_j'(y_j) \quad \text{and} \quad u'(c_j) \geq p \quad \text{for} \quad j = H, L, \tag{9}$$

so production and consumption are generally type-dependent.[4] If the consumer's constraint is not binding, then $u'(c_j) = p$, solved uniquely by $c(p) > 0$, so any unconstrained type spends $m^* = pc(p)$. If the constraint is binding, then $u'(c_j) > p$ so a type j consumes $c_j < c(p)$ and spends $m_j < m^*$. Thus,

$$c_j = \min\{\tfrac{m_j}{p}, c(p)\}. \tag{10}$$

The planner's allocation satisfies $u'(c_j) = \phi'_j(y_j)$, which is sustained only if $c_j = c(p)$, since $p = \phi'_j(y_j)$, i.e., when monetary constraints bind for no-one.

To find optimal savings of type j use (1) and (5) in (8) to obtain

$$V_j(m_j) = m_j + \tfrac{\alpha_j}{2}[u(c_j) - \phi_j(y_j)] + \tfrac{\alpha_j}{2}p(y_j - c_j) + W_j(0) \tag{11}$$

where c_j satisfies (10). If $m_j < m^*$ (constrained buyer), then $\frac{\partial c_j}{\partial m_j} = \frac{1}{p}$ and so

$$\tfrac{\partial V_j(m_j)}{\partial m_j} = 1 + \tfrac{\alpha_j}{2}\left[\tfrac{u'(c_j)}{p} - 1\right]. \tag{12}$$

The expected lifetime utility $V_j(m_j)$ depends on the agent's wealth m_j and two other elements: a type-dependent continuation payoff $W_j(0)$ and an expected surplus from market one trades. With identical probability $\frac{\alpha_j}{2}$ either the agent spends pc_j money enjoying utility $u(c_j)$, or earns py_j money suffering disutility $\phi_j(y_j)$. The change in wealth expected from market one trades, $p(y_j - c_j)$, is zero in a representative agent model since $y = c$. Instead, with unequal balances or productivity, produced and consumed amounts may be mismatched. Goods market clearing on odd dates implies

$$\alpha_H \rho y_H + \alpha_L(1-\rho)y_L = \alpha_H \rho c_H + \alpha_L(1-\rho)c_L. \tag{13}$$

[4] Since $\phi'' > 0$ then $y_j > 0$ for all j. With $\phi'' = 0$ only the most efficient type would produce.

Definition: *Given an initial money stock $\bar{M} > 0$ and a government policy (π, τ), a competitive stationary monetary equilibrium is a time-invariant list of real quantities $(c_j, y_j, q, x_{jk}, m_j)$ and prices $(p_{1,t}, p_{2,t})$ consistent with the government budget constraint (2), market clearing (3), (7) and (13), and optimality (9) and (10).*

In equilibrium, from $1 = \frac{\beta}{\pi} \times \frac{\partial V_j(m_j')}{\partial m_j'}$ and (12) one gets the Euler equation

$$\tfrac{\pi}{\beta} = 1 + \tfrac{\alpha_j}{2}\left[\tfrac{u'(c_j)}{p} - 1\right] \text{ for } j = H, L. \tag{14}$$

The right hand side displays the nominal yield on money, one, plus its expected liquidity premium. It is non-negative because money is needed to trade in market one and $u'(c_j) \geq p$ from (9). The liquidity premium grows with the severity of liquidity constraints and the probability of consumption shocks. The left hand side is the (gross) nominal interest rate on an illiquid bond (not traded here, but see Boel and Camera 2006), so let $i = \frac{\pi}{\beta} - 1$ be the net nominal interest rate. Since $p = \phi_j^{'}(y_j)$, then (14) is

$$i = \tfrac{\alpha_j}{2}\left[\tfrac{u'(c_j)}{\phi_j'(y_j)} - 1\right] \text{ for } j = H, L. \tag{15}$$

Hence, there are two equations in two unknowns (c_H, c_L), which can be uniquely determined as a function of the model's parameters and i, which summarizes the policy parameter in the present model. Hence, monetary policy affects consumption in market one.

Consider two classes of economies, with heterogeneity in trade shocks and in productivity. The former exhibits $\alpha_L < \alpha_H$ and $\phi_j(y) = \phi(y)$ for all j, hence $p = \phi^{'}(y)$

and output y_j is type-independent. The latter exhibits $\phi'_L(y) > \phi'_H(y)$ and $\alpha_j = \alpha$ for all j, hence $p = \phi'_j(y_j)$ and y_j is type-dependent.

Lemma: *Any stationary equilibrium must be such that $\pi \geq \beta$, i.e., $i \geq 0$. A unique stationary monetary equilibrium exists for $\pi > \beta$ and it is such that: (i) with trade shocks heterogeneity $m_L < m_H < m^*$, so $c_L < c_H < c(p)$; (ii) with productivity heterogeneity $m_j = m < m^*$ and $c_j = c < c(p)$ for all j; (iii) if $\pi \to \beta$ (the Friedman rule), then $m_j \to m^*$ and $c_j \to c(p)$ for all j.*

Proof. By way of contradiction, suppose a monetary equilibrium exists with $\pi < \beta$. From (14) one needs $\pi \geq \beta + \beta(\alpha_j/2)[u'(c_j)/\phi_j^{'}(y_j) - 1] \geq \beta$. This contradicts $\pi < \beta$. So, let $\pi > \beta$. From (14), as $\pi \to \beta$ then $u'(c_j) \to p = \phi_j^{'}(y_j)$, implying $c_j \to c(p)$ for $j = H, L$. Hence $m_H \to m^*$ and $m_L \to m^*$. Now consider trade shocks heterogeneity. By concavity of u, if $\pi > \beta$, then $u'(c_j) > p = \phi^{'}(y)$ for all j and so $c_L < c_H < c(p)$ and $m_L < m_H < m^*$. Consider productivity heterogeneity. Here $c_H = c_L = c$ since $\alpha_j = \alpha$ for all j in (14). Hence, $m_H = m_L = m < m^*$. If $\pi > \beta$, then $c < c(p)$, so $m < m^*$. Existence follows from inspection of optimality and market clearing conditions.■

The rate of return on money $\frac{1}{\pi}$ cannot exceed the shadow interest rate $\frac{1}{\beta}$ in steady state equilibrium. If that were the case, then agents would want to keep accumulating money, which is not a stationary monetary equilibrium. Second, the allocation is efficient as $i \to 0$ because $u'(c_j) = p = \phi_j^{'}(y_j)$ for all j. Individual money holdings in this case converge to the average value m^* because the liquidity premium vanishes, hence neither productivity nor trade-frequency differences affect saving decisions.

The equilibrium distribution of money depends on the heterogeneity considered. There is no equilibrium dispersion in money balances when agents differ only in productivity because trade shocks and preferences over goods are homogeneous, so agents self-insure and consume identically. However, $p = \phi_j'(y_j)$ so $y_L < y_H$, hence $x_{Hs} < x_{Ls}$ (from (4)). This means that low-productive agents work more than average in market two to make up for low market one sales. Instead, money balances are unequally distributed when trade shocks are heterogeneous because those more likely to trade self-insure more, holding more money than average. In this case inflation redistributes monetary wealth, as shown next.

Fixing π, let $c_{j\pi}$, $y_{j\pi}$, $m_{j\pi}$ and $\bar{m}_\pi$ denote equilibrium quantities, where (3) and (10) imply $\bar{m}_\pi = p[\rho c_{H\pi} + (1-\rho)c_{L\pi}]$ with $p = \phi_j'(y_{j\pi})$; use (6) and (11) to define equilibrium *ex-ante welfare* for type j by $V_{j\pi}$, where

$$\begin{aligned}(1-\beta)V_{j\pi} = \quad & \tfrac{\alpha_j}{2}[u(c_{j\pi}) - \phi_j(y_{j\pi})] + U(q^*) - q^* \\ & + \tfrac{\alpha_j}{2}\phi_j'(y_{j\pi})(y_{j\pi} - c_{j\pi}) + (\pi - 1)(\bar{m}_\pi - m_{j\pi}).\end{aligned} \tag{16}$$

Inflation π affects ex-ante welfare in three ways. It distorts market one consumption and output, hence it affects the expected trade surplus $\frac{\alpha_j}{2}[u(c_{j\pi}) - \phi(y_\pi)]$. This is the only distortion in a representative-agent setting, since the second line in (16) vanishes because $m_{j\pi} = \bar{m}$ and $c_{j\pi} = c_\pi = y_{j\pi} = y_\pi$ for all j. With heterogeneity, generally inflation affects $V_{j\pi}$ in two additional ways. It impacts *expected net earnings* in market one, $\frac{\alpha_j}{2}\phi_j'(y_{j\pi})(y_{j\pi} - c_{j\pi})$, which can be nonzero because agents may produce and consume different amounts. If money balances are heterogeneous, then inflation also redistributes monetary wealth thanks to inequalities in the *inflation tax* $(\pi - 1)(\bar{m}_\pi - m_{j\pi})$. Clearly, there is no redistribution if $\pi = 1$ (no inflation). If $\pi = \beta$, then the second line in (16) vanishes since $m_j \to m^* = \bar{m}$ and $c_{j\beta} \to c_\beta = y_\beta$ for all j. Instead, in the model with heterogeneous trade shocks inflation redistributes monetary wealth from the top to the bottom of the distribution, because $m_{L\pi} < \bar{m}_\pi < m_{H\pi}$ for all $\pi > \beta$. This mirrors the findings from the related matching models of Berentsen, Camera and Waller (2005), Chiu and Molico (2007a), and Molico (2006).

4 Quantitative analysis in the basic model

The welfare cost of inflation for a type j is a standard compensating variation measure. It is the percentage adjustment in consumption (both markets) that leaves the agent indifferent between some inflation $\pi > \beta$ and a lower rate $z \geq \beta$. Given that consumption is adjusted by the proportion $\bar{\Delta}_z$ (income, expenditure, and hours worked are unaltered), use (16) to define adjusted ex-ante welfare $\bar{V}_{jz}$ by

$$\begin{aligned}(1-\beta)\bar{V}_{jz} = \quad & \tfrac{\alpha_j}{2}[u(\bar{\Delta}_{jz}c_{jz}) - \phi_j(y_{jz})] + U(\bar{\Delta}_{jz}q^*) - q^* \\ & + \tfrac{\alpha_j}{2}\phi_j'(y_{jz})(y_{jz} - c_{jz}) + (z-1)(\bar{m}_z - m_{jz}).\end{aligned} \tag{17}$$

For a type j, the welfare cost of π instead of z inflation is the value $\Delta_{jz} = 1 - \bar{\Delta}_{jz}$ that satisfies $V_{j\pi} = \bar{V}_{jz}$. If $\Delta_{jz} > 0$, then type j is indifferent between π, or z inflation with consumption reduced by Δ_{jz} percent.

To calibrate common parameters and to compute benchmark measures for the welfare cost of inflation a representative-agent version of the model is considered. Then, heterogeneity is re-introduced. The focus is on a yearly model of the U.S. for the sample period 1929-2006. The nominal interest rate i is the annualized yield on short-term commercial paper, the nominal price level P is GDP deflator, aggregate nominal output PY is nominal GDP, and the nominal money supply M is $M1$.[5]

4.1 Representative-agent economy

Set $\alpha_j = \alpha$ and $\phi_j(y) = \phi(y)$ for all j, so $p = \phi'(y)$, $pc = m$ and $c = y$. Fix $u(c) = \frac{c^{1-a}-1}{1-a}$, $U(q) = A\ln(q)$ so $q^* = A$, and let $\phi(y) = \frac{y^\delta}{\delta}$. From (15) one gets

$$c = \left(\tfrac{\alpha}{2i+\alpha}\right)^{\frac{1}{\delta+a-1}} . \tag{18}$$

Set $\beta = 0.96$, $a = 1$ (i.e., $u(c) = \log c$), and $\delta = 1.1$.[6] The remaining parameters to calibrate are α and A. The procedure in Aruoba, Waller and Wright (2007) is used to calibrate α. First, the interest elasticity of $M1$ is estimated using a standard approach, obtaining -0.33756.[7] The theoretical interest elasticity of money demand

[5] For 1929-75, the yield on commercial paper is from Friedman and Schwartz (1982, Table 4.8, col. 6). For 1976-96, it is from *Economic Report of the President* (1996, Table B-69). For 1997-06, it is the Financial Commercial Paper with 3-month maturity in H.15 *Selected Interest Rates*, Federal Reserve Statistical release. $M1$ is in billions of dollars, December of each year, not seasonally adjusted. For 1929-58, it is from Friedman (1963, p. 708-718, col. 7). For 1959-06, it is from the St. Louis Fed *FRED Database*. For 1929-06, nominal GDP is from *The National Income and Product Accounts of the United States*. Running the analysis for a quarterly specification yields similar results (see Table 1); this matches the findings in Aruoba, Waller and Wright (2007). Additional sensitivity analyses and details on analytical derivations are in the working paper Boel and Camera (2009) and in the online appendix in Science Direct.

[6] This facilitates comparisons to studies based on Lagos and Wright (2005), which usually assume unit elastic preferences and linear disutility in both markets. Setting $\delta = 1$ has virtually no impact on our calibration and aggregate welfare cost results, but does not allow us to consider equilibria where differentially efficient producers are active.

[7] Following Goldfeld and Sichel (1990), the log of real money balances on each date t (M_t/P_t)

is $\varepsilon_m = \frac{2i\phi'(y)}{\alpha c u''(c)} = -\frac{2i}{(2i+\alpha)a}$. The average interest in the sample period is $i = 0.044$. Now, given $a = 1$, one finds that the value $\alpha = 0.145$ matches the theoretical to the empirical elasticity.

The parameter A is chosen to fit the ratio $L = \frac{M}{PY}$, which can be interpreted as money demand because real balances M/P are proportional to real output Y with a factor of proportionality $L(i)$ that depends on the nominal interest rate. For the empirical counterpart of L the above-described data is used. To construct the theoretical expression for L note that aggregate nominal output is $PY = p_1 \frac{\alpha}{2} c + p_2 A$, i.e., nominal output in markets one and two. From (3), the equilibrium nominal money stock $M = p_2 m$, so normalizing by p_2 one gets $L = \frac{m}{\frac{\alpha}{2} pc + A}$. Hence, $L = L(i) \equiv \frac{1}{\alpha/2 + Ac^{-\delta}}$, with c defined in (18). Given the parameters fixed above, the value $A = 2.537$ minimizes the distance between L in the data and in the model.[8] Figure 1 shows how the calibrated money demand (solid line) fits the data in the sample period (circles). The R^2 coefficient is 0.550. As a comparison, the dashed $L(i)$ is for a model where α is

is regressed on the date t log of real GDP, nominal interest rates, and one-period lagged balances: $\ln m_t = \gamma_0 + \gamma_1 \ln y_t + \gamma_2 \ln i_t + \gamma_3 \ln m_{t-1} + v_t$. To account for first-order autocorrelation in the residuals v_t the Cochrane-Orcutt procedure is used.

[8] The parameter $\alpha = 0.145$ may seem "small," since some studies set $\alpha = 1$ to minimize the search frictions (Lagos and Wright, 2005, Chiu and Molico 2007a) or calibrate α to higher values (Aruoba, Waller, and Wright, 2007, Chiu and Molico 2007b). However, the fit of the model worsens for $\alpha > 0.145$ ($\alpha = 1$ gives the poorest fit). In addition, though α has no obvious empirical counterpart, it affects the share of market one output, $\frac{\alpha}{2} L(i)$. In our model this share is bounded above by 13% (set $\alpha = 1$ and calibrate $A = 2.537$); in the calibrated model it is about 2%. Similar shares emerge from other studies; in Aruoba, Waller and Wright (2007) the share is less than 10% (around 4% in the calibrated model), in Chiu and Molico (2007a) it is below 9%, and it is below 10% in Lagos and Wright (2005) at 4% inflation. This suggests the calibrated parameter α is not too small.

selected to deliver the best possible fit.[9]

Figure 1 and Table 1 approximately here

Table 1 reports the welfare cost of 10% anticipated inflation as opposed to no inflation and the Friedman rule. These costs are around or below 1% of consumption, in line with the findings from studies based on various representative-agent models.[10] The next sections study how heterogeneity affects this initial finding.

4.2 Heterogeneous trade shocks

Suppose agents differ only in trade shocks. As seen earlier, equilibrium money holdings are heterogeneous, $m_{L\pi} < \bar{m}_\pi < m_{H\pi}$ and the parameters $(\rho, \alpha_L, \alpha_H)$ pin down the shares of money held by different segments of society. Given the parameters fixed above, let $\alpha = 0.145$ correspond to average trade shocks, i.e., $\rho\alpha_H + (1-\rho)\alpha_L = 0.145$, and calibrate $(\rho, \alpha_L, \alpha_H)$ using U.S. data on the distribution of liquidity holdings. The *Survey of Consumer Finances* of the Federal Reserve Board reports a measure called "liquidity," which includes the total value of all types of transactions accounts held by surveyed U.S. households. Dividing households into income quintiles, the share

[9] The parameter A is calibrated for α values going from 0.025 to 1. The coefficient R^2 rises quickly with α, attains a maximum $R^2 = .61$ for $\alpha \approx .075$, and then drops slowly to .15. The implied share of market one output rises in α. Intuitively, the best fit requires a sufficiently small share of monetary trade. Chiu and Molico (2007b) obtain a remarkable fit by including endogenous costly participation in market two; unfortunately, this reduces analytically tractability, so this case is not considered in this study.

[10] For example, the welfare cost of 10% inflation (as opposed to no inflation) is around 1.3% in Lagos and Wright (2005) and 0.7% in Aruoba, Waller and Wright (2007), for similar pricing mechanisms; it is around 1% in Lucas (2000) and just a fraction of 1% in Cooley and Hansen (1989). See also the discussion and references in Lucas (2000) and Lagos and Wright (2005).

of liquidity held in 1995 by the top two quintiles of U.S. households was 94.1%, while the bottom 60% held the remaining liquidity (shares are not dramatically different in earlier years). In the model $\bar{m}$ is the theoretical measure of total liquidity, so $\frac{\rho m_H}{\bar{m}}$ is the share of liquidity held by types H. Associating $j = H$ to the top two income quintiles gives $\rho = 0.4$. The values $(\alpha_L, \alpha_H) = (0.003, 0.357)$ match the theoretical liquidity share to its empirical counterpart.

The average (or aggregate) welfare cost of inflation in this heterogeneous-agent version of the model remains positive, though it is smaller than for the representative agent (Table 1). The reason is that the burden of inflation is now unevenly distributed. Those who consume less hold less money than average and suffer less because the inflation tax redistributes to them some of the monetary wealth of the richer agents. To check the sensitivity of the results consider mean-preserving spreads for (α_L, α_H) arbitrarily fixing $\rho = 0.5$ and varying α_L from 0 to 0.145. Figure 2 reports the welfare costs (average and type-specific) against α_L; moving left to right equilibrium consumption and wealth disparities fall, converging to the representative-agent model.

Figure 2 approximately here

To sum up the results, in this heterogeneous-agent model anticipated inflation lowers aggregate welfare, but the burden of inflation falls mostly (or solely) on the shoulders of the high-consumption, 'rich' segment of society. The aggregate welfare loss is smaller than in the representative-agent model and is affected by monetary wealth inequality. Wealth disparities result in unequal inflation tax burdens, which induce a top-to-bottom redistribution of monetary wealth. This redistribution reduces the welfare loss of the poor and, in fact, can even increase their welfare, which is why in Figure 2 the average welfare cost falls with greater heterogeneity. However, inflation is never beneficial to society as a whole, i.e., the positive redistributive effect does not dominate the consumption distortions so $i = 0$ is always the best policy.

The welfare cost of inflation for a given segment of society increases with the share of monetary wealth held by that segment. In Figure 2 the welfare cost for agent j rises with α_j because m_j rises. Redistributive effects are stronger the greater is the disparity in monetary wealth, which is why inflation benefits no-one when there is little dispersion in money holdings (far right in Figure 2).

The above findings share similarities and differences with results from related models that exhibit nondegenerate equilibrium monetary distributions, e.g., Chiu and Molico (2007a,b), Molico (2006), and Reed and Waller (2006), as well as dissimilar models, e.g., Akyol (2004), Erosa and Ventura (2002), and Imrohoroglu (1992). On the one hand, one can draw a parallel between the quantitatively small societal welfare loss from anticipated inflation in the present work and other works. The welfare cost of 10% inflation is close to zero in Akyol (2004), around 0.6% in Chiu and Molico (2007a,b), 1.57% in Erosa and Ventura (2002), about 1% in Imrohoroglu (1992), and around 1% (relative to the Friedman rule) in Reed and Waller (2006).

On the other hand, differences emerge from comparing other results, especially those regarding implications for optimal monetary policy and the redistributive impact of inflation. First, the present study suggests that inflation's redistributive impact mitigates the overall welfare loss but is not a sufficient reason to run any policy other than zero nominal interest rates, i.e., moving away from the Friedman rule cannot generate societal welfare gains. This is unlike in Molico (2006), where some inflation can raise welfare, or the precautionary balances model in Akyol (2004) where small welfare gains are also possible.[11] Second, the top-to-bottom direction of monetary wealth redistribution is unlike in Erosa and Ventura (2002) where, given increasing returns to scale in the cost to liquidate high-return assets, inflation can act as a *regressive* tax.

4.3 Heterogeneous productivity

Now suppose agents have identical needs for consumption insurance but different labor productivity. Fix the preference parameters to the calibrated representative-agent values and give differently efficient production technologies to different agent types. A type L must supply $\theta - 1$ more hours than a type H to produce the same amount of output y, i.e., $\phi_j(y) = \frac{(\theta_j y)^\delta}{\delta}$ with $\theta_L = \theta > \theta_H = 1$. Interpret $\theta_j y_j$ as hours worked to produce y_j output, i.e., type L agents must work longer than type H to produce the same amount of output. Hence $\phi_L(y) > \phi_H(y)$ for all $y > 0$. With this formulation one can define (c, y_L, y_H) as explicit functions of the parameters and since

[11] In Molico (2006) agents can self-insure only at random, which is why low inflation can improve average welfare. Instead, in our model and Chiu and Molico (2007a,b) self-insurance opportunities arise deterministically. In Akyol (2004) inflation redistributes income top-to-bottom.

choosing output or hours worked is equivalent, we y_j is used instead of hours.

The relative productivity parameter θ is calibrated to match the ratio of productivity in the service sector (very productive) to the goods sector (less productive). Productivity is measured by average output per hour in nonfarm private industries using data from the Bureau of Labor Statistics for 1987-2006. Hence, $\theta = 4.24$. Then, fix $\rho = 0.77$ to match the proportion of employment in the service sector.

The welfare cost of inflation in this heterogeneous economy is unequally distributed, with the (more) productive agents suffering the most (Table 1). The average welfare cost is very close to that for the representative agent since there is neither equilibrium dispersion in money holdings (inflation cannot redistribute wealth) nor in consumption (consumption distortions are identical across agents). Welfare cost disparities stem from inequality in market one average net earnings (that sum up to zero). Productive agents earn more than they spend on average, $y_{H\pi} > c_j > y_{L\pi}$, and their income falls with inflation. So, the social burden of inflation lies mostly on their shoulders.

Unlike the previous heterogeneous-agents version, no segment of this unequally productive society benefits from inflation (Table 1). The reason is that by fixing $\delta = 1.1$, the model implies a large wage elasticity of labor supply in market one (it is $\frac{1}{\delta-1}$). To determine how the wage elasticity impacts the results, Figure 3 reports the welfare cost of inflation for $\delta \in [1.01, 5]$, i.e., wage elasticities falling from 100 to 0.25. As the elasticity falls, the welfare cost falls and, for a sufficiently low elasticity, it turns negative for the less productive. This is equivalent to income redistribution. Hence, even in this model with no equilibrium monetary wealth inequality, inflation can benefit low-consumption agents at the expense of high-consumption agents.

Figure 3 approximately here

5 Money is not the only asset

Money is typically the only financial asset available in the class of models to which this study belongs.[12] However, the impact of inflation on social welfare should depend on whether alternative assets exist that can provide consumption insurance and offer some inflation protection. So, the model is extended to let agents hold more "sophisticated" financial portfolios.

To induce equilibrium heterogeneity in financial portfolios set $\alpha_L < \alpha_H$ and fix $\phi_j = \phi$ for all j. To augment financial sophistication, introduce a prototypical competitive financial sector that offers risk-pooling services. In market two agents can buy consumption insurance from an intermediary selling one-period nominal assets at price $\theta > 0$. Assets can only be redeemed in the following market one for claims to money, which are enforceable in market two and are financed with the revenue from asset sales. The intermediary earns zero profits and operates at zero resource cost.

In this version of the model money and assets offer some consumption insurance, and trade frictions affect financial markets, also. Market one buyers can redeem the

[12] But see Bencivenga and Camera (2008), Lagos and Rocheteau (2008), or Telyukova and Wright (2007).

asset and spend its claims to consume. Sellers can redeem the asset to cash its claims in the next market. However, idle agents cannot participate in market one, i.e., can access neither goods nor financial markets and so cannot redeem the asset. This form of limited participation in financial and goods markets affects agent types differently. The asset is less attractive to those who are less likely to be present on market one.

For a type j holding $b_j \geq 0$ assets and $m_j \geq 0$ money one must add b_j and $\pi\theta b_j'$ to the right hand sides in, respectively, (1) and (4). Hence,

$$V_j(m_j, b_j) = m_j + \alpha_j b_j + \tfrac{\alpha_j}{2}[u(c_j) - \phi(y)] + \tfrac{\alpha_j}{2}p(y - c_j) + W_j(0,0), \tag{19}$$

where $pc_j \leq m_j + b_j$, so $c_j = \min\{\frac{m_j+b_j}{p}, c(p)\}$. Clearly $\frac{\partial V_j(m_j,b_j)}{\partial b_j} = \alpha_j + \frac{\alpha_j}{2}[u'(c_j) - p]\frac{\partial c_j}{\partial b_j}$ where $\frac{\partial c_j}{\partial b_j} = \frac{1}{p}$ for a constrained buyer. As usual, the agent's need for consumption insurance depends on α_j. Equation (14) is still needed for $m_j > 0$, while $b_j \geq 0$ if

$$\theta\tfrac{\pi}{\beta} \geq \alpha_j + \tfrac{\alpha_j}{2}\left[\tfrac{u'(c_j)}{p} - 1\right]. \tag{20}$$

As done earlier, consider stationary outcomes where all market one buyers are constrained. Note that wealthier U.S. households have less liquid and more sophisticated financial portfolios than those at the bottom of the wealth distribution (Erosa and Ventura 2002). So, conjecture an outcome in which those who consume less than average hold more money but less assets than average. The simplest scenario is $b_H > b_L = 0$ and $m_L > m_H = 0$. It is optimal if for $j = L$, then (14) holds and (20) is a strict inequality; the converse must hold for $j = H$. This is an equilibrium for some sufficiently small inflation rate bounded away from β.

To demonstrate it observe that if only types H buy πb assets at price θ, then the repayment constraint faced by the intermediary is

$$\pi\theta b = \alpha_H b, \tag{21}$$

which gives the price θ consistent with zero profits. Since α_H is the redemption probability for $j = H$, the asset's expected return is $\frac{\alpha_H}{\theta}$ and it equals the inflation rate. In this sense, the asset can insure types H against inflation.

From (20)-(21), one sees that $b_H > 0$ requires

$$\alpha_H(\tfrac{1}{\beta} - 1) = \tfrac{\alpha_H}{2}\left[\tfrac{u'(c_H)}{p} - 1\right]. \tag{22}$$

Notice that $\alpha_H(\frac{1}{\beta} - 1) < \frac{\pi}{\beta} - 1$ for all $\pi > \bar{\pi} = \beta + \alpha_H(1-\beta) \in (\beta, 1)$. If (22) holds, then $\frac{\pi}{\beta} - 1 > \frac{\alpha_H}{2}\left[\frac{u'(c_H)}{p} - 1\right]$ for all $\pi > \bar{\pi}$ (so $m_H = 0$). As $\pi \to \beta$ then $u'(c_H) = p$ (efficiency) and type H holds only money. Intuitively, if $\pi \le \bar{\pi}$, then inflation is small and assets offer consumption insurance that is too expensive relative to the insurance offered by money. Otherwise, type H agents prefer holding assets but not money, since by doing so they can consume more.

Now consider a type L. Optimality implies $b_L = 0$ and $m_L > 0$ when $\pi < \tilde{\pi} = \beta + \alpha_H - \beta\alpha_L$; note that $\tilde{\pi} > \bar{\pi}$ and $\tilde{\pi} > 1$ if $\beta > \frac{1-\alpha_H}{1-\alpha_L}$.[13] Intuitively, when $\pi < \tilde{\pi}$ assets offer consumption insurance that is too expensive for agents who trade less frequently than average. These agents place less value on the asset and buy it only if inflation is

[13] From (14), optimality requires $\frac{\pi}{\beta} - 1 = \frac{\alpha_L}{2}\left[\frac{u'(c_L)}{p} - 1\right]$ for $m_L > 0$. For $b_L = 0$ expression (20) must hold as a strict inequality. This occurs if $\theta\frac{\pi}{\beta} > \alpha_L + \frac{\pi}{\beta} - 1$. Use (21) to get $\pi < \tilde{\pi}$.

sufficiently high, i.e., if money is a sufficiently poor store of value.

To sum up, if $\pi \in (\bar{\pi}, \tilde{\pi})$, then only types L hold money, so those who have the most money are not the ones who consume and trade the most. Hence, $(c_L, c_H) = (\frac{m_L}{p}, \frac{b}{p})$ satisfy (14) and (22), $(m_L, m_H) = (\frac{\bar{m}}{1-\rho}, 0)$, and $(b_L, b_H) = (0, b)$. Now

$$W_j(m_{j,k}) = U(q_j) - q_j - \pi\theta b_j' - \pi m_j' + m_{j,k} + \tau + \beta V_j(b_j', m_j'), \tag{23}$$

which differs from (5) due to asset holdings.

Using (19) and (23), equilibrium ex-ante welfare for a type j is

$$\begin{aligned}(1-\beta)V_j(b_j, m_j) = \quad & \tfrac{\alpha_j}{2}[u(c_j) - \phi(y)] + U(q^*) - q^* \\ & + \tfrac{\alpha_j}{2}p(y - c_j) + (\pi - 1)(\bar{m} - m_j) + b_j(\alpha_j - \pi\theta).\end{aligned} \tag{24}$$

Here, $b_j = pc_j - m_j$ and $p = \phi'(y)$. The term $b_j(\alpha_j - \pi\theta)$ captures the impact of inflation on asset holdings. Given $m_H = b_L = 0$, the net inflation tax is $-(\pi - 1)\frac{\rho}{1-\rho}\bar{m}$ for type L and $(\pi - 1)\bar{m}$ for H. So, inflation generates a wealth transfer from L to H types. Assets holdings are not subject to the inflation tax because the expected return on assets is π, i.e., the asset price perfectly adjusts for inflation.[14]

Given the calibrated parameters one gets $(\bar{\pi}, \tilde{\pi}) = (0.975, 1.315)$. Hence, a comparison is made between equilibria with 0% and 10% inflation in which only type L agents hold money. Inflation still generates a positive average welfare cost. However, the impact is quantitatively smaller and the redistributive effects of inflation are reversed, compared to the money-only version of the model (Table 1). Now it is the poor who would pay to avoid inflation, while the wealthy would demand *more* consumption.

[14] Since $m_L = pc_L$ the last two terms in (24) are $(\pi - 1)\rho pc_L$ and $(\pi - 1)(1 - \rho)pc_L$ for $j = L, H$.

Inflation lowers societal welfare less than before because not everybody holds money in this version of the model. The redistributive impact of inflation is reversed because those who trade less frequently not only save and consume less than average, but are also the only ones who save with money. Hence the burden of inflation falls entirely on the shoulders of the poorest segment of society.

The basic lesson is that the economy's financial structure not only affects the size of the welfare loss imposed by inflation on society, but it can also have significant consequences for how this loss is distributed across society. Whether inflation is more a concern for the rich or for the poor depends on whether agents are differentially able to participate in goods *and* financial markets. In the model, those who have greater need for consumption insurance can also more easily participate in financial markets.

6 Final remarks

This study has considered a monetary economy where ex-ante heterogeneous agents hold money to insure against consumption risk. Stationary equilibrium exhibits tractable forms of dispersion in monetary wealth and earning profiles. By calibrating the model to the U.S. economy, it has been shown that the societal welfare loss from moderate anticipated inflation is not large. Yet, the impact of inflation can vary noticeably across society. If money is unequally distributed in equilibrium and it is the only asset, then inflation can benefit low-consumption agents by redistributing monetary wealth top-to-bottom. The direction of redistribution can change if additional assets exist that provide consumption insurance, because wealthier agents might prefer to hold less money than poorer agents. The lesson is that the burden of inflation can be unequally distributed across society, not only depending on frictions in trade but also on the financial structure of the economy. Camera, Chiu, and Molico (2009) takes this analysis a step further with a model capable of generating richer distributions of wealth and money.

References

[1] Akyol, A., 2004. Optimal monetary policy in an economy with incomplete markets and idiosyncratic risk. Journal of Monetary Economics 51 (6), 1245-1269.

[2] Albanesi, S., 2007. Inflation and inequality. Journal of Monetary Economics 54 (4), 1088-1114.

[3] Aliprantis, C.D., Camera, G., Puzzello, D., 2007. Anonymous markets and monetary trading. Journal of Monetary Economics 54 (7), 1905-1928.

[4] Andolfatto, D. (2009). On the societal benefits of illiquid bonds. Unpublished working paper, Simon Fraser University

[5] Aruoba, S., Waller, C.J., Wright, R., 2007. Money and capital. Working Paper 07-14, FRB of Cleveland.

[6] Bencivenga, V., Camera, G., 2007. Banks, liquidity insurance, and interest on reserves in a matching model of money. Unpublished manuscript.

[7] Berentsen, A., Camera, G., Waller, C.J., 2005. The distribution of money balances and the non-neutrality of money. International Economic Review 46 (2), 465-487.

[8] Bhattacharya J., Haslag, J.H., Martin, A., 2005. Heterogeneity, redistribution, and the Friedman rule. International Economic Review 46 (2), 437-454.

[9] Boel, P., Camera, G., 2006. Efficient monetary allocations and the illiquidity of bonds. Journal of Monetary Economics 53 (7), 1693-1715.

[10] Boel, P., Camera, G., 2009. Financial sophistication and the distribution of the

welfare cost of inflation. Working Paper, Purdue University.

[11] Camera, G., Chiu, J., Molico, M., 2008. A search model with illiquid bonds. Unpublished paper. Bank of Canada.

[12] Chiu, J., Molico, M., 2007a. Idiosyncratic uncertainty, inflation, and welfare. Unpublished paper. Bank of Canada.

[13] Chiu, J., Molico, M., 2007b. Liquidity, redistribution, and the welfare cost of inflation. Working Paper 2007-39, Bank of Canada.

[14] Cooley, T.F., Hansen, G.D., 1989. The inflation tax in a real business cycle model. American Economic Review 79 (4), 733-748.

[15] Economic Report of the President, 1996. Govt. Printing Office, Washington D.C.

[16] Erosa, A., Ventura, G., 2002. On inflation as a regressive consumption tax. Journal of Monetary Economics 49 (4), 761-795.

[17] Friedman, M., 1963. A Monetary History of the United States 1867-1960. Princeton University Press, Princeton.

[18] Friedman, M., Schwartz, A.J., 1982. Monetary Trends in the United States and the United Kingdom, 1867-1975. University of Chicago Press, for the National Bureau of Economic Research, Chicago.

[19] H.15 Selected Interest Rates, Federal Reserve Statistical Release.

[20] Goldfeld, S., Sichel, D.E., 1990. The demand for money. In: Friedman, B.M., Hahn, F.H. (Eds.), Handbook of Monetary Economics. Elsevier, Amsterdam, pp.

299-356.

[21] Imrohoroglu, A., 1992. The welfare cost of inflation under imperfect insurance, Journal of Economic Dynamics and Control 16(1), 79-91.

[22] Lagos, R., Rocheteau, G., 2008. Money and capital as competing media of exchange. Journal of Economic Theory 142(1), 247-258.

[23] Lagos, R., Wright, R., 2005. A unified framework for monetary theory and policy analysis. Journal of Political Economy 113 (3), 463-484.

[24] Lucas, R., 2000. Inflation and welfare. Econometrica 68 (2), 247-274.

[25] Molico, M., 2006. The distribution of money and prices in search equilibrium. International Economic Review 47 (3), 701-722.

[26] Reed R., Waller, C.J., 2006. Money and risk sharing. Journal of Money, Credit and Banking 38 (6), 1599-1618.

[27] Telyukova, I.A., Wright, R., 2007. A model of money and credit, with application to the credit card debt puzzle. Review of Economic Studies 75 (2), 629-247

[28] U.S. Bureau of the Census, 1960. Historical statistics of the United States, colonial times to 1957. Govt. Printing Office, Washington D.C..

[29] U.S. Office of Business Economics. The National Income and Product Accounts of the United States. Govt. Printing Office, Washington D.C..

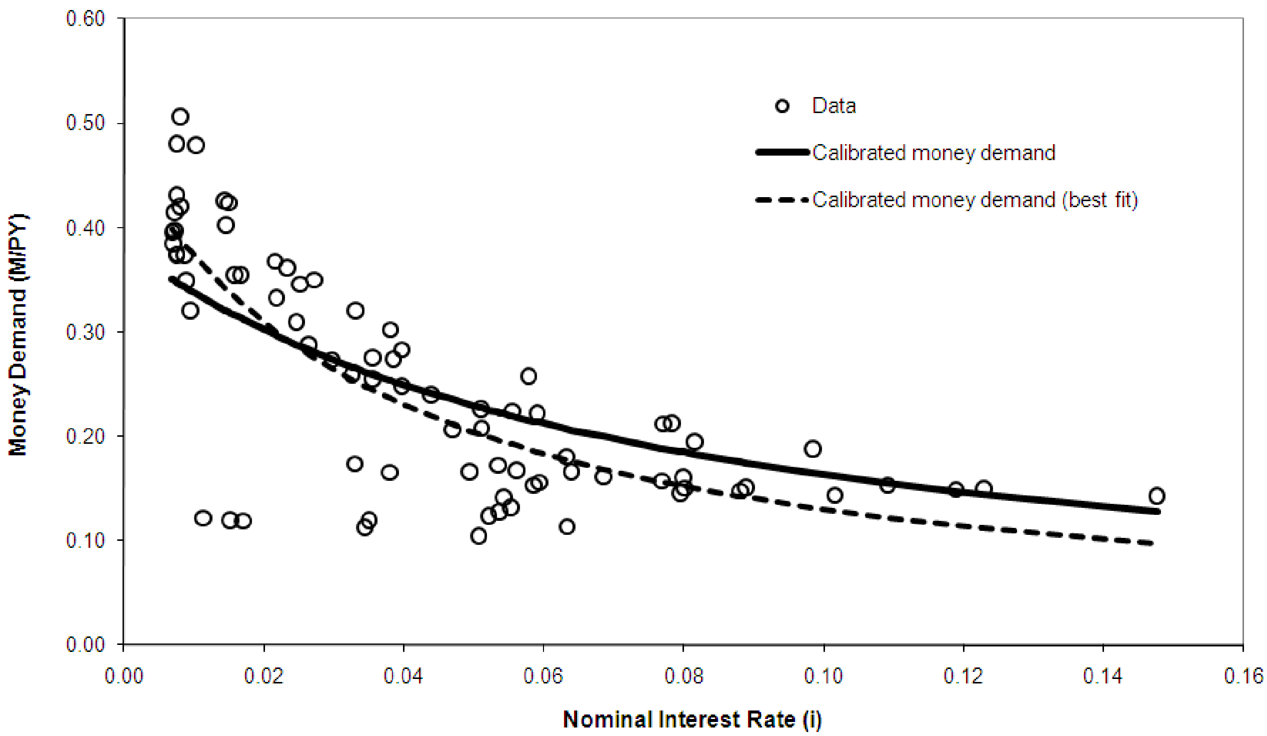

Figure 1: US money demand with fitted model.

Notes: Each circle identifies *M/PY* against *i*, for each year in the sample period 1929-2006. The solid line depicts the calibrated money demand *L(i)*. The dashed line depicts a calibrated money demand for a model where α is selected to deliver the best possible fit.

	No Inflation			Friedman Rule		
	Average	**Top 40%**	**Bottom 60%**	**Average**	**Top 40%**	**Bottom 60%**
Representative agent	0.868 (0.668)	--	--	1.077 (0.828)	--	--
Het. trade shocks	0.300 (0.222)	1.932 (1.547)	-0.787 (-0.662)	0.429 (0.327)	2.042 (1.629)	-0.646 (-0.541)
Het. trade shocks + asset	0.044 (0.037)	-0.023 (-0.021)	0.089 (0.077)	0.134 (0.079)	-0.006 (-0.021)	0.227 (0.146)
Het. productivity	0.868 (0.669)	0.891 (0.690)	0.790 (0.600)	1.078 (0.830)	1.128 (0.873)	0.911 (0.684)

Table 1: Percentage welfare cost of 10 percent inflation relative to No Inflation and the Friedman rule.

Notes: Results for a quarterly specification of the model are in parentheses. Quarterly data are for the period 1947- 2006. M1 is seasonally adjusted and for each quarter we consider M1 from the third month of the quarter. For the period 1947-1958, M1 is from Friedman's *A Monetary History of the United States, 1857-1960*. For the period 1959-2006 it is from the FRED database at the St. Louis Fed. Output is annualized GDP from the U.S. BEA (quarterly data), so we divide each data point by 4. The price is GDP deflator from the U.S. BEA. The interest rate is the annualized yield of the 3 month T-bill from FRED (monthly data). To get a quarterly interest rate, we average the monthly data for each quarter and divide this average value by 4. The discount rate is now 0.01 so β= 0.99, and 10% annual inflation rate implies π-1=2.41% in the quarterly specification.

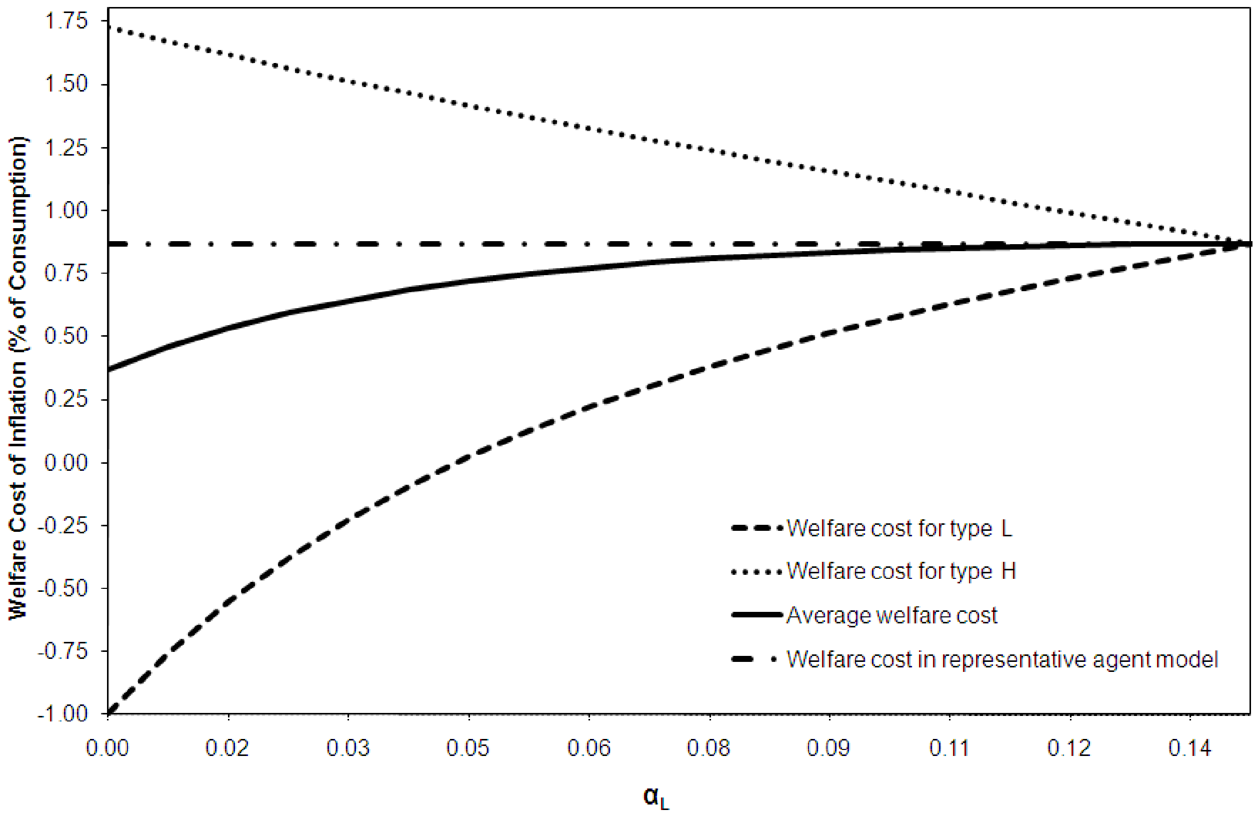

Figure 2: Percentage welfare costs of 10 percent inflation, relative to no inflation, against α_L

Notes: The figure is drawn for the model with heterogeneity in trade risk, so that $\alpha_L < \alpha_H$. The average value of the parameters α_j is given the calibrated value $\alpha=0.145$ in the representative model given $\rho=0.5$.

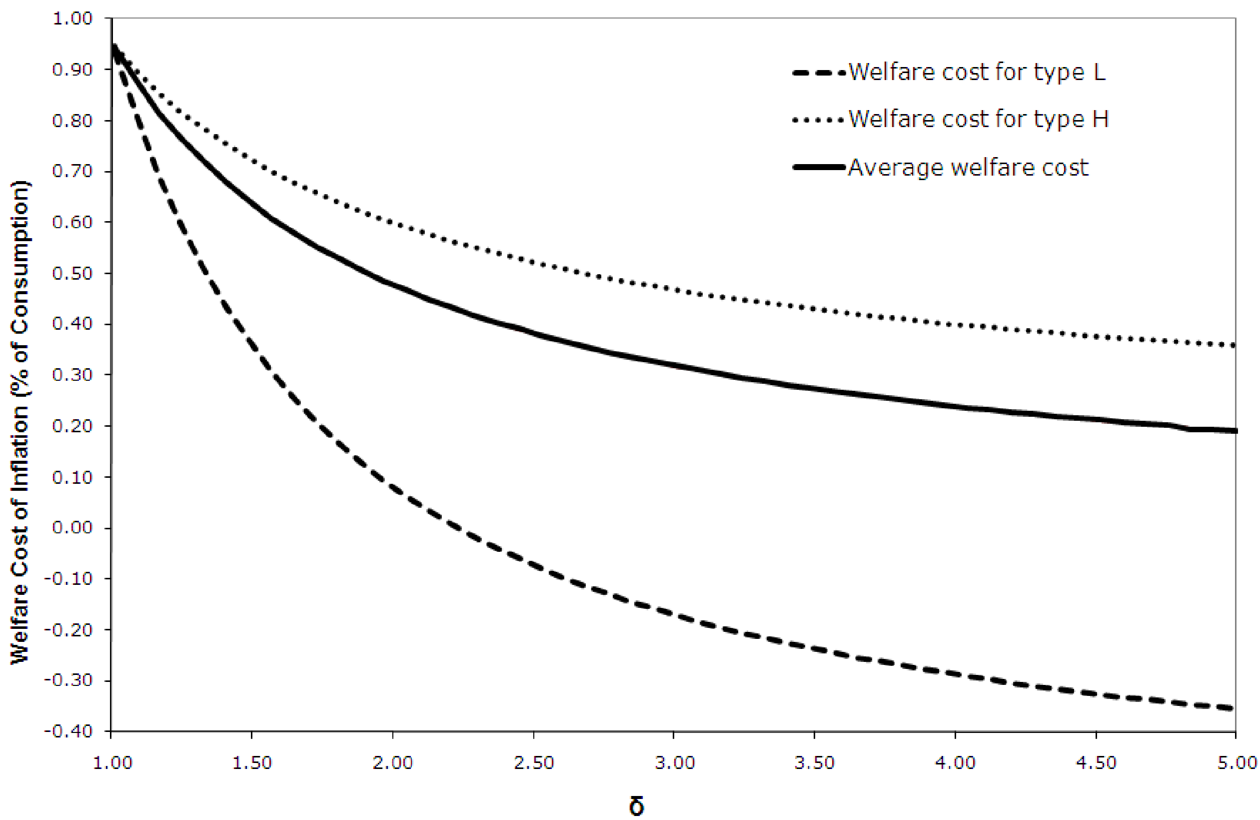

Figure 3: Percentage welfare costs of 10 percent inflation, relative to no inflation, against δ.

Notes: The figure is drawn for the model with heterogeneity in market one productivity. The relative productivity parameter is θ=4.24 and the proportion of H types is ρ=0.77. The wage elasticity falls as δ increases. As δ varies α and *A* are not recalibrated because ε_m is independent of δ and *L(i)* is also independent of δ because *a*=1. The welfare cost curve for the representative agent model overlaps with the average welfare cost curve reported in the figure.

Appendix A: Helpful Derivations

A.1 The constrained-efficient allocation

Consider the allocation selected by a planner who maximizes the agents' lifetime utilities and treats agents identically. The planner is subject to the same physical and informational constraints faced by the agents and therefore cannot observe identities. However, the planner observes types. Basically, the planner can propose a type-dependent consumption plan in each trade cycle, but does not have the ability to transfer resources across agents over time. Equivalently, the planner maximizes expected utility of the arbitrary agent on each date. The planning problem thus corresponds to a sequence of static maximization problems, i.e., to maximizing ex-ante welfare of the representative agent, subject to technological feasibility.

Recall that on each date agents have identical preferences ex-ante and there is an identical proportion of buyers and sellers. Moreover, on each odd date agents that are active can produce or consume with equal probability.

Letting $\rho_j = \rho$ for $j = H$ and $1 - \rho$ for $j = L$, the planner problem is to choose $\{c_j, y_j\}_{j=H,L}$, q, and x to solve:

$$\begin{aligned} \max \quad & \textstyle\sum_{j=H,L} \frac{\alpha_j}{2} \rho_j [u(c_j) - \phi_j(y_j)] + U(q) - x \\ \text{s.t.} \quad & \textstyle\sum_{j=H,L} \rho_j c_j \leq \sum_{j=H,L} \rho_j y_j \quad \text{and } q \leq x \end{aligned}$$

By non-satiation, the feasibility constraints should hold with equality. Letting λ denote the Lagrange multiplier on the first feasibility constraint, the FOCs are thus

$$\tfrac{\alpha_j}{2}\rho_j[u'(c_j)-\lambda]=0$$

$$\tfrac{\alpha_j}{2}\rho_j[-\phi_j'(y_j)+\lambda]=0$$

$$U'(q)-1=0$$

That is, agents produce up to the point where the marginal utility of their consumption or labor equal the marginal utility of income, λ.

Hence, the efficient allocation is stationary across trade cycles, and it can be characterized as follows. On odd dates $c_j = c^* = \rho y_H + (1-\rho)y_L$ and $y_L = y_L^* < y_H = y_H^*$ where the starred output values are the unique positive solutions to the two equalities $u'(y_L + y_H) = \phi_j'(y_j)$ for $j = H, L$. It should be clear that $c^* = y^*$ such that $u'(c^*) = \phi'(c^*)$ if there is no heterogeneity in productivity. On even dates $q_j = x_j = q^*$ for each type j in each trade cycle, where q^* is the unique positive solution to $U'(q) = 1$.

A.2 Optimal choices in market one

The optimal choice $y_j \geq 0$ of a type-j producer must satisfy $\phi_j'(y_j) \geq \frac{\partial W_j(m_{j,s})}{\partial m_{j,s}}\frac{\partial m_{j,s}}{\partial y_j}$. The optimal c_j of a type-j buyer must satisfy $u'(c_j) + \frac{\partial W_j(m_{j,b})}{\partial m_{j,b}}\frac{\partial m_{j,b}}{\partial c_j} \geq 0$, omitting the multiplier on his budget constraint. Clearly, $\frac{\partial W_j(m_{j,k})}{\partial m_{j,k}} = 1$ and $\frac{\partial m_{j,s}}{\partial y_j} = -\frac{\partial m_{j,b}}{\partial c_j} = p$ from

$$m_{j,b} = m_j - pc_j, \quad m_{j,s} = m_j + py_j, \quad \text{and} \quad m_{j,n} = m_j. \tag{1}$$

Hence, one gets $p \leq \phi_j'(y_j)$ and $u'(c_j) \geq p$ for $j = H, L$.

Elasticities and the money demand ratio L

Consider a representative agent economy and focus on odd dates.

Elasticity of disutility of labor. The disutility of labor is $\phi(y) = \frac{y^\delta}{\delta}$, where y is production as well as labor effort. So, the elasticity of disutility of labor is

$$\varepsilon_y = \frac{d\phi(y)/\phi(y)}{dy/y} = \frac{d\ln\phi(y)}{d\ln y} = \frac{y^{\delta-1}y}{y^\delta}\delta = \delta,$$

since the differential $d\ln\phi(y) = d\ln(y^\delta/\delta) = d(\delta\ln y - \ln\delta) = \frac{\delta}{y}dy$. Since $\phi'(y) = p$, the labor supply $y(p)$ satisfies

$$y^{\delta-1} = p \Rightarrow y(p) = p^{\frac{1}{\delta-1}}.$$

Elasticity of labor supply. In our model the wage of a worker on odd dates is p. The elasticity of the labor supply with respect to the relative wage is

$$\varepsilon_p = \frac{dy(p)/y(p)}{dp/p} = \frac{d\ln y(p)}{d\ln p} = \frac{1}{\delta-1},$$

because the differential

$$d\ln y(p) = d(\ln p^{\frac{1}{\delta-1}}) = d\left(\frac{1}{\delta-1}\ln p\right) = \frac{1}{\delta-1} \times \frac{dp}{p}.$$

Elasticity of money demand. From

$$c_j = \min\{\tfrac{m_j}{p}, c(p)\}, \tag{2}$$

one gets $pc = m$, so the Euler equation

$$i = \frac{\alpha_j}{2}\left[\frac{u'(c_j)}{\phi_j'(y_j)} - 1\right] \text{ for } j = H, L, \tag{3}$$

for the representative agent gives

$$F(m/p, i) = \frac{\alpha}{2}\left[\frac{u'(m/p)}{\phi'(y)} - 1\right] - i = 0.$$

Using the implicit function theorem we have

$$\frac{\partial m/p}{\partial i} = -\frac{\partial F/\partial i}{\partial F/\partial(m/p)} = -\frac{-1}{\frac{\alpha}{2\phi'(y)}u''(m/p)} = \frac{2\phi'(y)}{\alpha u''(m/p)}.$$

Given $c = m/p$ and market clearing $c = y$, the elasticity of money demand is

$$\varepsilon_m = \frac{\partial m/p}{\partial i} \times \frac{i}{m/p} = \frac{2\phi'(y)}{\alpha u''(c)} \times \frac{i}{c} = \frac{2i\phi'(y)}{\alpha c u''(c)} \tag{4}$$

We have $\phi'(y) = y^{\delta-1}$ and $y = c$. So (4) is $\frac{2ic^{\delta-1}}{\alpha c u''(c)}$. Substituting c from

$$c = \left(\frac{\alpha}{2i+\alpha}\right)^{\frac{1}{\delta+a-1}} \tag{5}$$

one gets

$$\varepsilon_m = -\frac{2i}{a(2i+\alpha)}.$$

The money demand ratio L. $L = \frac{m}{\frac{\alpha}{2}pc+A}$ and from (2) we have $pc = m$. Also, $p = \phi'(y)$. Since $\phi'(y) = y^{\delta-1}$ and $y = c$ from market clearing, then $L = \frac{1}{\alpha/2+Ac^{-\delta}}$, with c defined in (5) as a function of parameters and interest rate.

A.3 Explicit solutions for consumption and output

Heterogeneity in trade risk. Here $y_H = y_L = y$. Given the assumed functional forms we have $\phi'(y) = y^{\delta-1}$ and $u'(c_j) = c_j^{-a}$ so rewrite the Euler equation (3) as $1 + \frac{2i}{\alpha_j} = \frac{c_j^{-a}}{y^{\delta-1}}$ for $j = H, L$, which implies $c_L = \left[\frac{(2i+\alpha_L)\alpha_H}{\alpha_L(2i+\alpha_H)}\right]^{-\frac{1}{a}} c_H$. From market clearing

$$\alpha_H \rho y_H + \alpha_L(1-\rho)y_L = \alpha_H \rho c_H + \alpha_L(1-\rho)c_L \tag{6}$$

one gets

$$y = \frac{\rho\alpha_H c_H + (1-\rho)\alpha_L c_L}{\rho\alpha_H + (1-\rho)\alpha_L}.$$

Substituting for y and c_L in the Euler equation above

$$c_H = \left\{ \frac{\alpha_H}{2i+\alpha_H} \left[\frac{\alpha_H\rho+\alpha_L(1-\rho)\left(\frac{(2i+\alpha_L)\alpha_H}{\alpha_L(2i+\alpha_H)}\right)^{-\frac{1}{a}}}{\alpha_H\rho+\alpha_L(1-\rho)} \right]^{1-\delta} \right\}^{\frac{1}{a+\delta-1}}.$$

Heterogeneity in productivity. From Lemma 2 in the paper we have $c_H = c_L = c$. Given the assumed functional forms $\phi_j'(y_j) = \theta_j^\delta y_j^{\delta-1}$ and $u'(c) = c^{-a}$ so rewrite the Euler equation as $1 + \frac{2i}{\alpha} = \frac{c^{-a}}{\theta_j^\delta y_j^{\delta-1}}$ for $j = H, L$. From market clearing (6) we have $c = \rho y_H + (1-\rho)y_L$; from $p = \phi_j'(y_j)$ for $j = H, L$, we have $p = \phi_H'(y_H) = \phi_L'(y_L)$, which is

$$y_H = y_L \left(\frac{\theta_L}{\theta_H}\right)^{\frac{\delta}{\delta-1}} = y_L\theta^{\frac{\delta}{\delta-1}}$$

since we have normalized $\theta_L = \theta > \theta_H = 1$. So, market clearing implies $c = y_L\left(\rho\theta^{\frac{\delta}{\delta-1}} + 1 - \rho\right)$. Substituting for c in the Euler equation above

$$y_L = \left[\left(1 + \frac{2i}{\alpha}\right)\left(\rho\theta^{\frac{\delta}{\delta-1}} + 1 - \rho\right)^a \theta^\delta\right]^{\frac{1}{1-a-\delta}}.$$

Money is not the only asset. Here $y_H = y_L = y$. The expression for c_L is obtained from

$$\frac{\pi}{\beta} = 1 + \frac{\alpha_j}{2}\left[\frac{u'(c_j)}{p} - 1\right] \text{ for } j = H, L, \tag{7}$$

and c_H is obtained from

$$\alpha_H(\tfrac{1}{\beta} - 1) = \frac{\alpha_H}{2}\left[\frac{u'(c_H)}{p} - 1\right]. \tag{8}$$

Given the assumed functional forms $\phi'(y) = y^{\delta-1}$ and $u'(c_j) = c_j^{-a}$ so the Euler equation (7) is $1 + \frac{2i}{\alpha_L} = \frac{c_L^{-a}}{y^{\delta-1}}$. From (8) one gets $\frac{2-\beta}{\beta} = \frac{c_H^{-a}}{y^{\delta-1}}$ which implies

$$c_L = \left(\frac{\alpha_L+2i}{\alpha_L} \times \frac{\beta}{2-\beta}\right)^{-\frac{1}{a}} c_H.$$

We have $y = \frac{\rho\alpha_H c_H + (1-\rho)\alpha_L c_L}{\rho\alpha_H + (1-\rho)\alpha_L}$ from (6). Substituting for y and c_L in (8) one gets

$$c_H = \left\{ \frac{\beta}{2-\beta} \left[\frac{\alpha_H\rho + \alpha_L(1-\rho)\left(\frac{\alpha_L+2i}{\alpha_L}\frac{\beta}{2-\beta}\right)^{-\frac{1}{a}}}{\alpha_H\rho + \alpha_L(1-\rho)} \right]^{1-\delta} \right\}^{\frac{1}{a+\delta-1}}.$$

Given $m_H = b_L = 0$ we have $(1-\rho)m_L = \bar{m}$ and $b = pc_H$. For a type L one has

$$(1-\beta)V_L(0,\bar{m}) = \frac{\alpha_L}{2}[u(c_L) - \phi(y)] + U(q^*) - q^* + \frac{\alpha_L}{2}p(y - c_L) - (\pi - 1)\frac{\rho}{1-\rho}\bar{m}.$$

Since $\bar{m} = (1-\rho)m_L = (1-\rho)pc_L$ then $(\pi - 1)\bar{m}\frac{\rho}{1-\rho} = (\pi - 1)\rho pc_L$. For a type H,

$$(1-\beta)V_H(b,0) = \frac{\alpha_H}{2}[u(c_H) - \phi(y)] + U(q^*) - q^* + \frac{\alpha_H}{2}p(y - c_H) + (\pi - 1)\bar{m}$$

because $\pi\theta = \alpha_H$, $m_H = 0$ and $b_H = b = pc_H$.

Appendix B: Model Fit

In this Appendix we present a rudimentary discussion about the fit of the theoretical money demand to the data for various model specifications.

1 Variations in preference parameters

In this section we calibrate market one preferences parameters in a different manner.

1.1 Linear disutility of labor in market one

If we set $\delta = 1$ as in Lagos and Wright (2005) and related papers, then the fit is virtually identical to the one we obtain with the specification found in the paper; the calibrated parameters α, A, and R^2 do not vary. Clearly, we need some convexity in disutility for coexistence of efficient/inefficient producers in the heterogeneous version of the model, hence we use $\delta = 1.1$.

1.2 Variations in a

Suppose now that we move away from unit elastic preferences in market one. For example suppose the parameter a is set to 0.71 to match a recent empirical study on risk aversion in Raj (2006).[1] In this case we get $\alpha = 0.248$, $A = 2.618$ and the fit falls

[1] Raj, C. (2006). A new method of estimating risk aversion. *American Economic Review*, 96 (5), 1821-1834.

slightly relative to our current model, $R^2 = 0.520$. So, there is not much difference in the fit. The welfare cost calculations do not change very much, either.

1.3 Variations in δ

Now suppose that, in addition to fixing $a = 0.71$, we also vary the disutility of labor in market one to match data on labor elasticities for the U.S.. Notice that δ corresponds to the elasticity of disutility of labor with respect to labor effort in market one. The elasticity of the labor supply with respect to the relative wage is $\frac{1}{\delta-1}$. So, set δ to match average elasticity of labor supply with respect to own wage in the U.S.. Estimates of the elasticity of labor supply vary according to the group considered (e.g., male versus female). From Filer, Hamermesh, and Rees (1996)[2] estimates of labor supply elasticities are 0.00 for men and 0.80 for women. Consequently, we set δ to match the average of the two values with weights given by the proportion of men (0.55) and women (0.45) in the labor force for the period 1960-2006 as reported by the Bureau of Labor Statistics. We get $\delta = 3.78$. Now fix $(a, \delta) = (0.71, 3.78)$. We obtain $\alpha = 0.248, A = 2.801$, and the fit falls to $R^2 = 0.460$. Figure B1 illustrates how the model fits the data.

[2] Filer, R. K., D. S. Hamermesh, and A. E. Rees (1996). *The Economics of Work and Pay*, Sixth Edition. New York: Harper-Collins.

1.4 Using off-the-shelf estimates of elasticity of money demand

Still, fix $(a, \delta) = (0.71, 3.78)$. We can calibrate the model to match an off-the-shelf elasticity of money demand, instead of using our own estimate to see how the model performs. For example, suppose we consider theelasticity in Aruoba, Waller and Wright (2007), who consider a different sample period for the U.S. and obtain an estimated elasticity of money demand of -0.226. In this way we obtain $\alpha = 0.427$ and $A = 3.052$. The fit is poorer because R^2 is 0.328. Figure B2 illustrates the fit of the model to the data.

2 Quarterly specification

Suppose instead we use a quarterly specification of our model. Hence, fix $(a, \delta) = (1, 1.1)$. We pin α down to match our estimate of the yearly elasticity of money demand (i.e., -0.3376). This implies $\alpha = 0.041$, $A = 0.781$, and $R^2 = 0.400$. So, the fit is worse than for a yearly model. Figure B3 illustrates the fit of the model to the quarterly data. With trade shock heterogeneity $\alpha_L = 0.001$ and $\alpha_H = 0.101$. The percentage of output produced in market one for this calibration is slightly lower than in the calibration presented in the paper. The upper bound on the share of market one output is not much different than the specification reported in the paper. The welfare costs are reported in Table 1 in the paper.

3 The trading friction α and the fit of the implied money demand

We have run the following sensitivity analysis for the representative agent model. Suppose we fix α, i.e., we do not calibrate it specifically to match the elasticity of money demand. Given α, choose A to match the empirical money demand, as usual. What is the α value that generates the best possible fit? How does the fit change with α? How does the share of market one output vary with α? The result is in Figure B4. It shows (horizontal axis) α versus R^2, and the ratio of output produced in market one to overall output.

The best possible fit is obtained for $\alpha = 0.075$, a value even smaller than the one that matches the estimated yearly elasticity of money demand. This measure of fit is hump-shaped in α. This means that if the model assigns too much or too little importance to monetary trade (market one trade is exclusively monetary, unlike market two transactions), then the implied money demand fits the data very poorly. See Figures B5-B6.

The calibrated value $\alpha = 0.145$ generates a fit close to the best possible fit (R^2 is 0.55 vs. 0.61). Greater values of α result in an even worse fit. To best match the empirical money demand the model should exhibit a sufficiently small share of monetary trade (out of total trade). In this sense the calibrated value 0.145 of the trade parameter α is not too small.

Now consider the share of market one output. It rises in α and we know from previous work that this share should not be too large (e.g., see Aruoba, Waller and Wright, 2007). Even in this sense, our trade parameter is not too small.

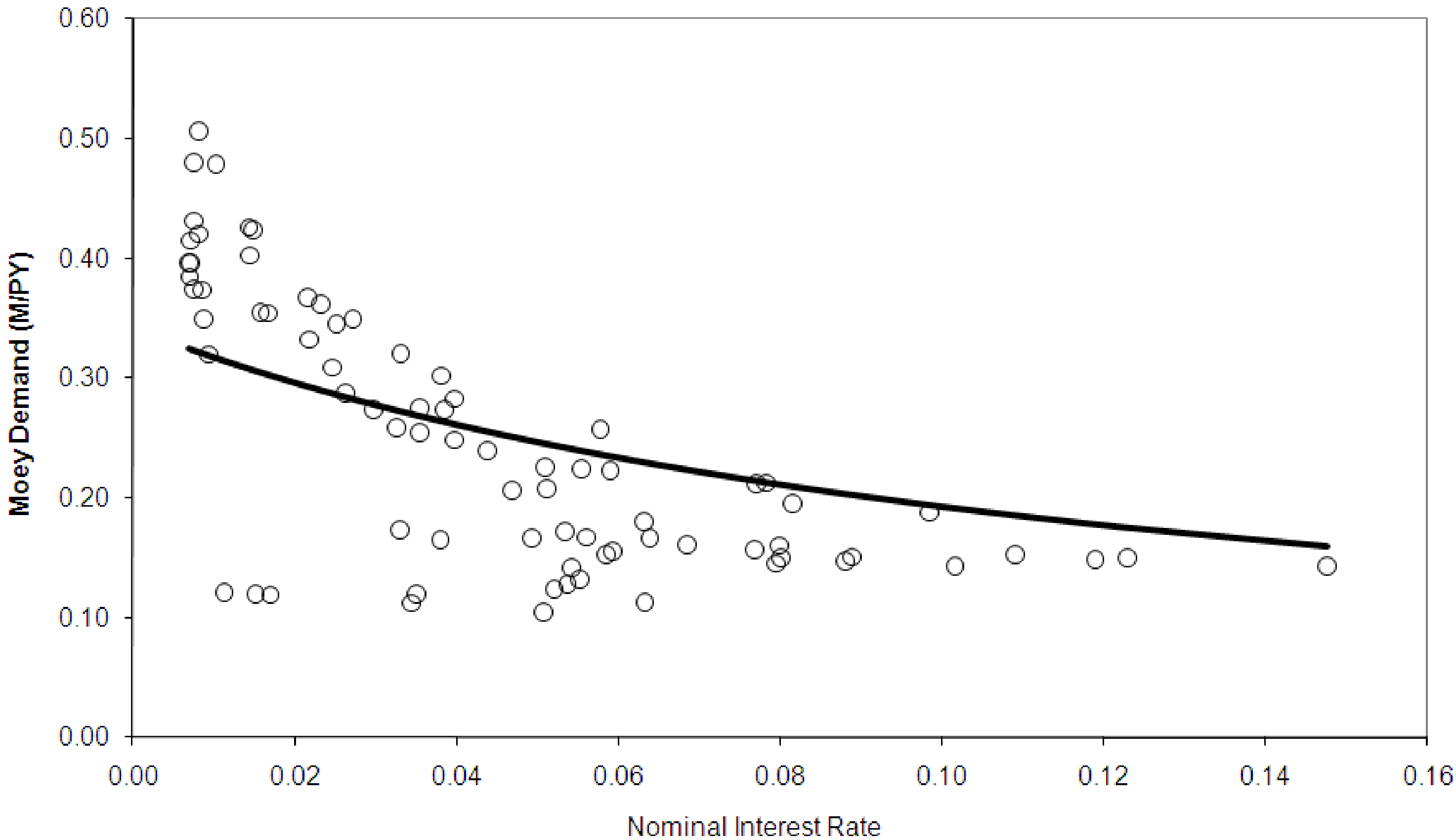

FIGURE B1: fit for α=0.71 and δ=3.78

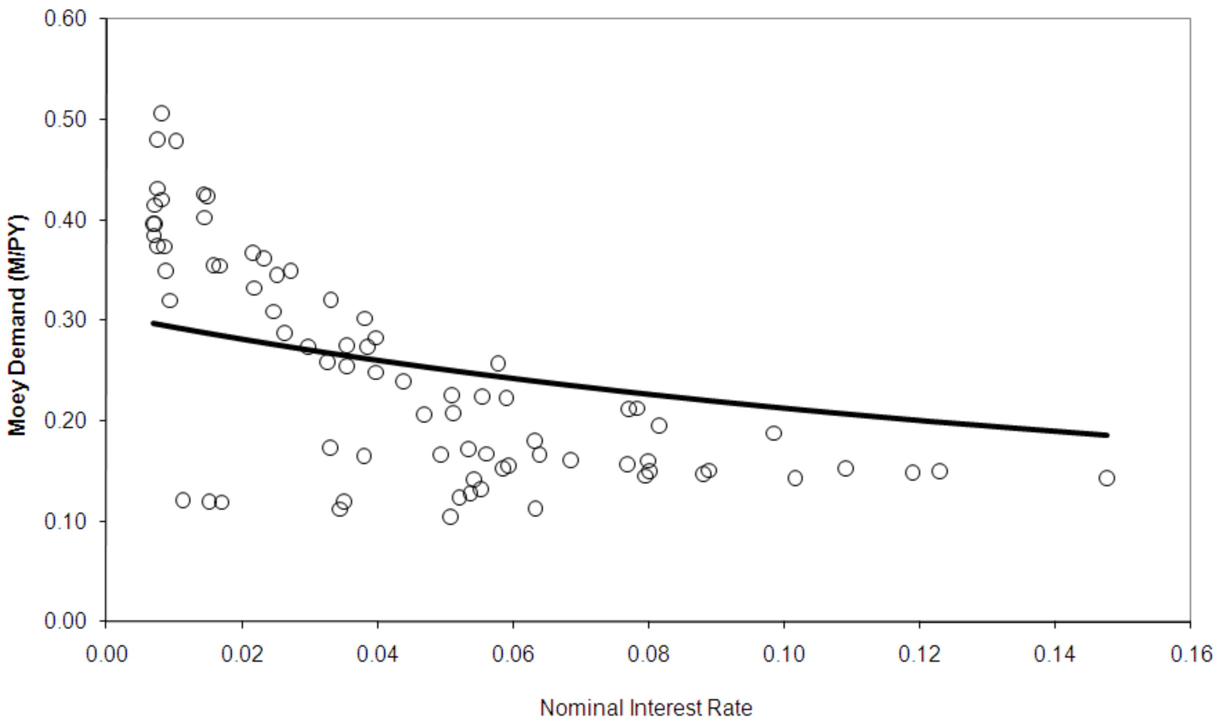

FIGURE B2: fit for α=0.71, δ=3.78 and A calibrated to elasticity from AWW 2007

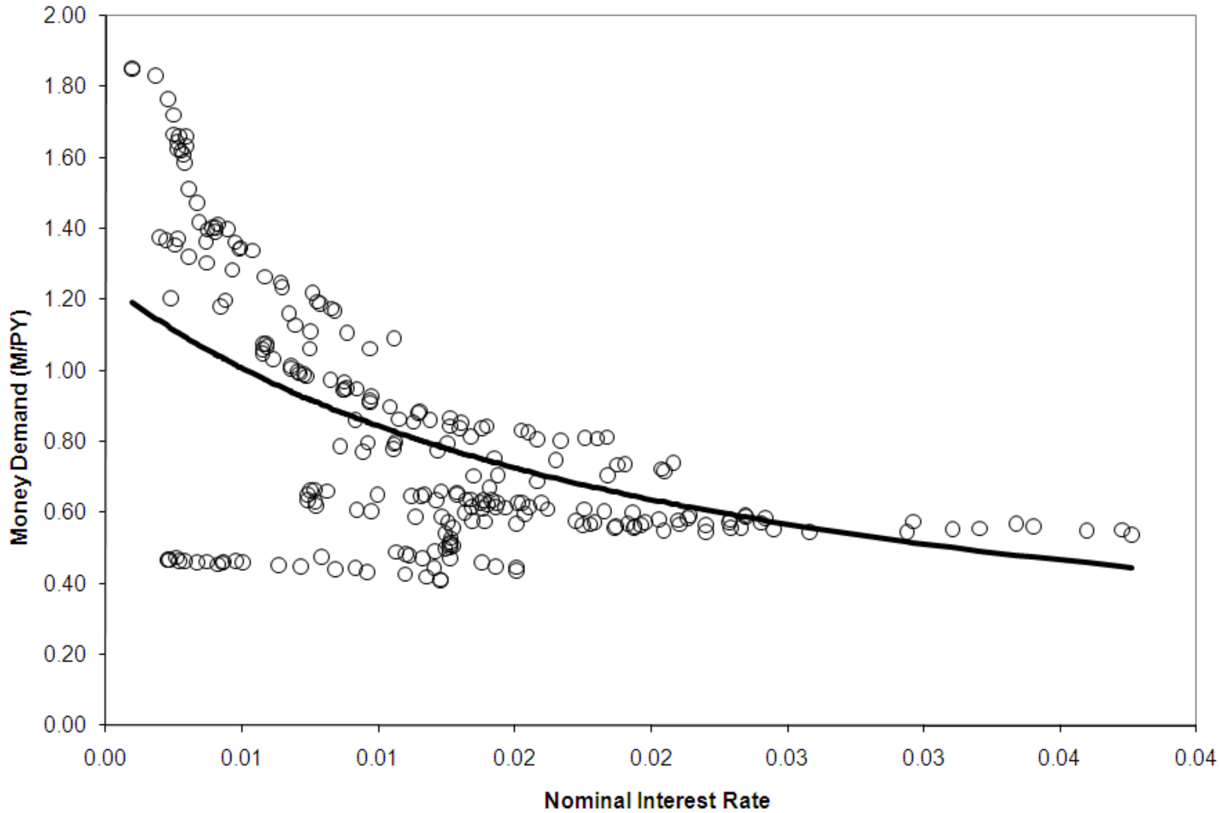

FIGURE B3: fit for quarterly specification

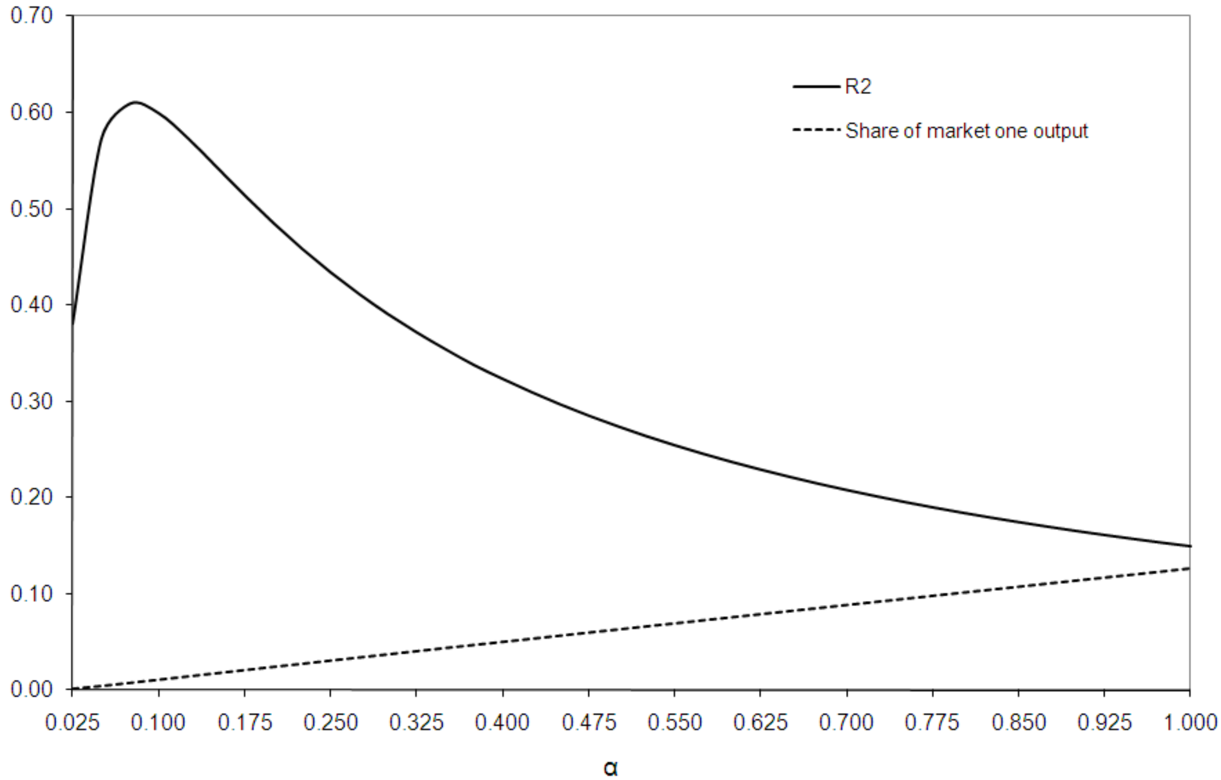

FIGURE B4: R2 and share of market one output as a function of α

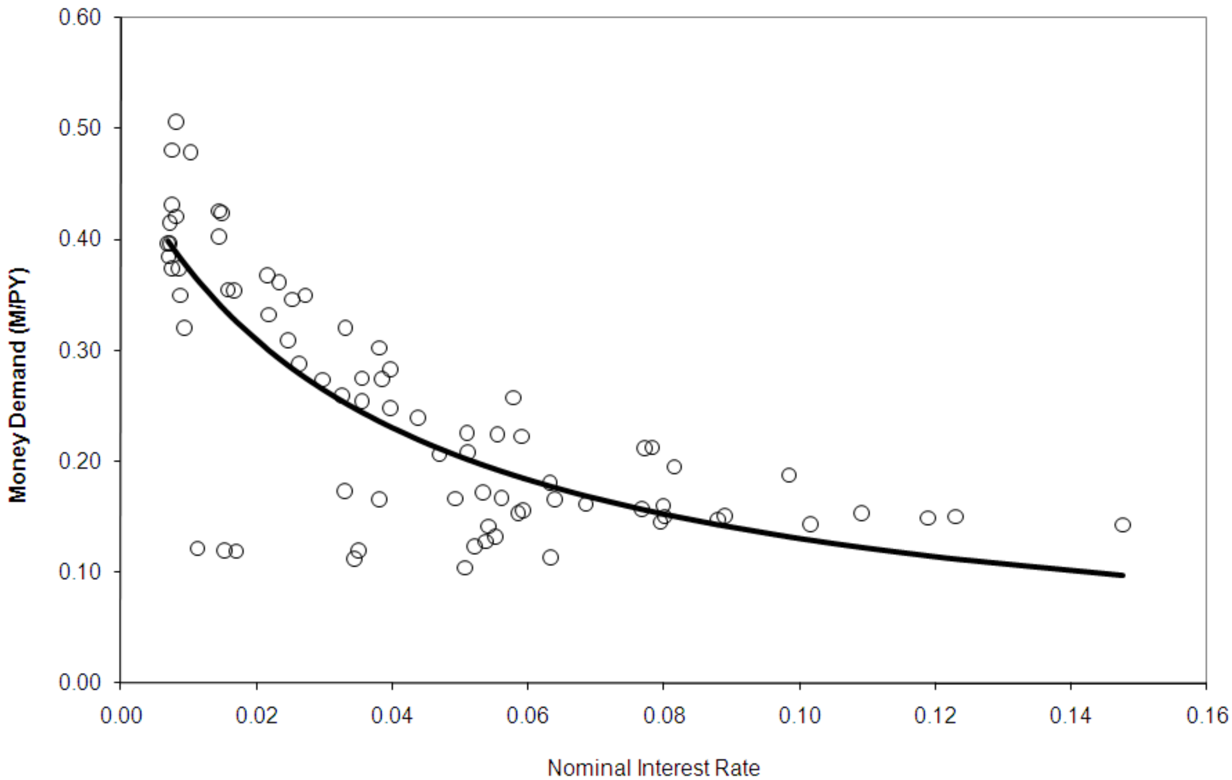

FIGURE B5: fit for parameter specification with highest fit (α=0.075, A=2.084, R^2 =0.61)

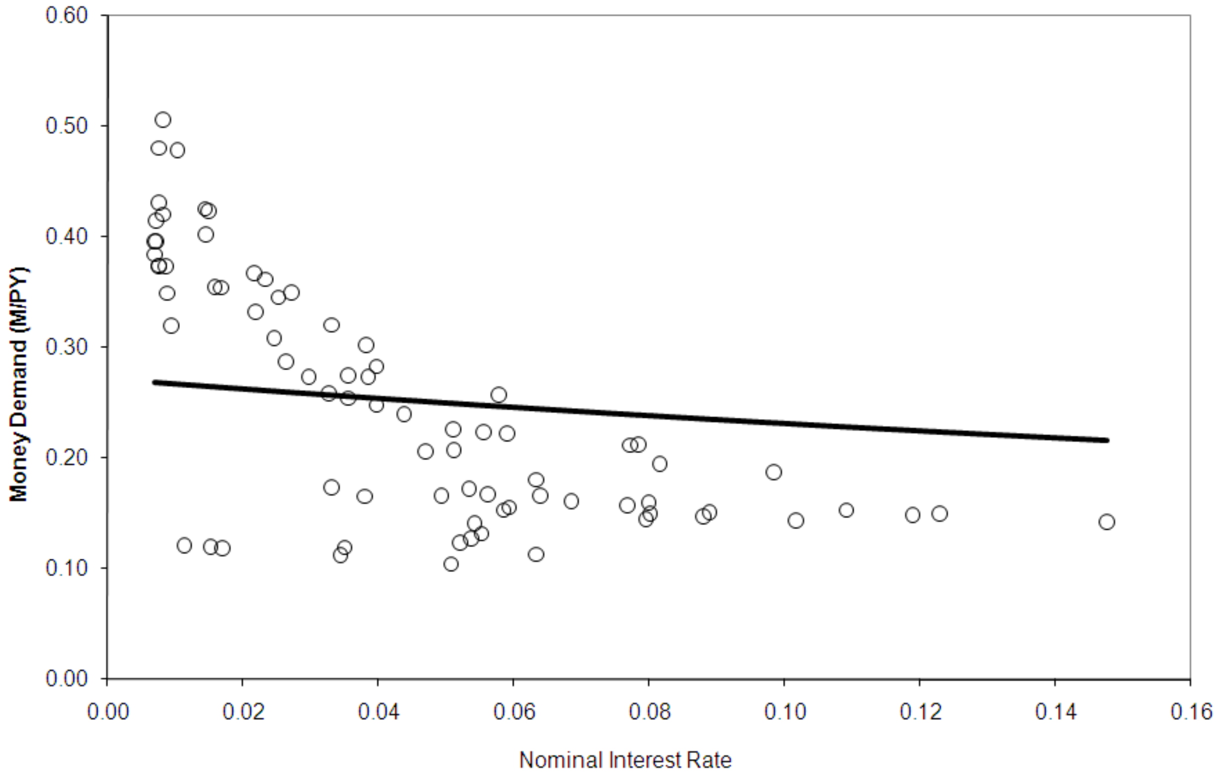

FIGURE B6: fit for parameter specification with lowest fit (α=1, A=3.176, R^2 =0.15)