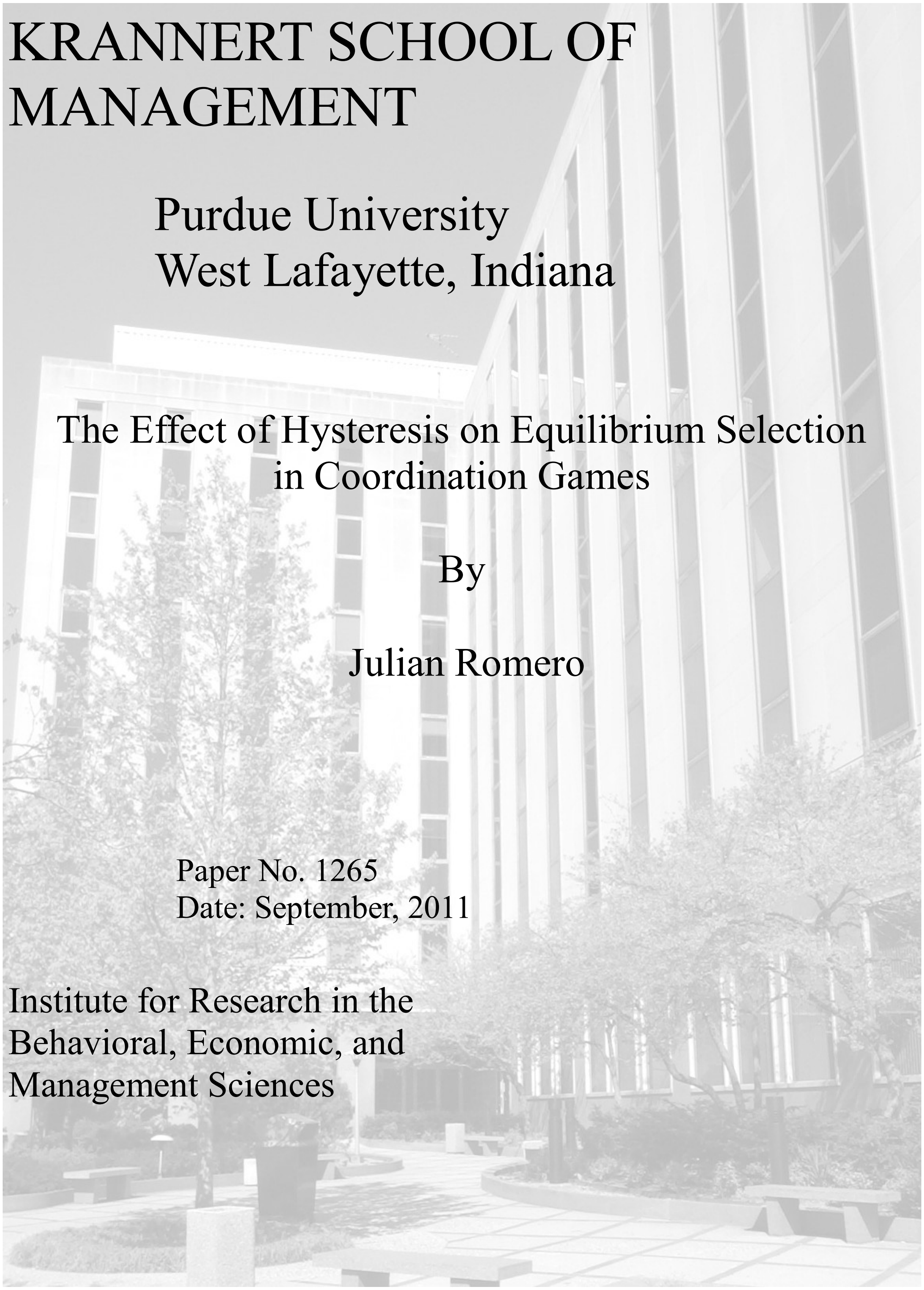

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The Effect of Hysteresis on Equilibrium Selection in Coordination Games

By

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The Effect of Hysteresis on Equilibrium Selection in Coordination Games*

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Abstract:

One of the fundamental problems in both economics and organization is to understand how individuals coordinate. The widely used minimum-effort coordination game has been used as a simplified model to better understand this problem. This paper first presents some theoretical results that give conditions under which the minimum-effort coordination game exhibits hysteresis. Using these theoretical results, some experimental hypotheses are developed and then confirmed using human subjects in the laboratory. The main insight is that play in a given game is heavily dependent on the history of parameters leading up to that game. For example, the experiments show when cost $c = 0.5$ in the minimum-effort coordination game, there is significantly more high effort if the cost has increased to $c = 0.5$ compared to when the cost has decreased to $c = 0.5$. One implication of this is that a temporary change in parameters may be able move the economic system from a bad equilibrium to a good equilibrium.

JEL classification: *C72, C92, M53*

Keywords: Hysteresis, Minimum-effort Coordination Game, Logit Equilibrium, Experimental Economics, Equilibrium Selection

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1 Introduction

One common cause of sub-optimal economic outcomes is the inability of individuals to coordinate their actions. For example, there are many situations where individuals could mutually benefit if they were able to coordinate their actions, but the environment makes coordination difficult. When faced with these situations, it is important to understand what aspects of the environment make coordination difficult, and try to determine what can be done to help individuals attain better outcomes. This paper focuses on hysteresis, or path dependence, and the effect it has on equilibrium selection in coordination games. The paper starts with a simple model, and develops theoretical results describing conditions under which hysteresis occurs in coordination games. Using the theoretical results, some experimental hypotheses are formulated, and are then confirmed using laboratory experiments with the minimum-effort coordination game. These results suggest that temporary changes in the environment can lead to a significant increase in the amount of coordination.

A system is said to exhibit hysteresis if there is path dependence. For example, in the context of a minimum-effort coordination game with cost parameter c, the system is said to exhibit hysteresis if the level of effort is significantly different at $c = 0.5$ depending on whether the cost has increased to $c = 0.5$ or decreased to $c = 0.5$. To better understand this hysteresis, this paper looks at a theoretical model of the minimum-effort coordination, and finds conditions under which this phenomena is likely to occur. In particular, this paper focuses on s-shaped equilibrium correspondences, because these correspondences lead to hysteresis (further details in Section 2).

Using the theoretical results from this model, some experimental hypotheses are developed and then tested. The experiments involve subjects playing the minimum-effort coordination game repeatedly as the cost is varied. There are two treatments, one in which the cost is varied from low to high to low, and another where it is varied form high to low to high. Varying the cost in this organized manner provides a test for determining whether hysteresis occurs in the minimum-effort coordination game. The experiments provide strong support for hysteresis in the minimum-effort coordination game showing large differences in effort levels at $c = 0.5$ depending on whether cost has increased or decreased to $c = 0.5$. Specifically, groups chose high effort 85% of the time when c started low and increased to 0.5 compared to 13% of the time when c started high and decreased to 0.5. Another important implication of hysteresis supported by the experimental results is that a temporary decrease in the cost parameter may move the system from a bad equilibrium to a good equilibrium. In the experiments, 13% of groups played the high effort at $c = 0.5$, but after the cost was temporarily decreased and then changed back to $c = 0.5$, the level of groups playing high effort increased significantly to 69%. These results highlight the implications of hysteresis on equilibrium selection.

One approach to better understanding the equilibrium selection problem is to impose additional equilibrium criteria, which refines the set of equilibria and makes predictions more clear. In their seminal work, Harsanyi and Selten (1988) suggest two different methods for selecting equilibria in games with multiple equilibria: "payoff dominance" and "risk dominance". In certain situations, these two selection criteria may conflict, meaning the payoff dominant equilibrium is not the risk dominant equilibrium. When there is conflict, Harsanyi and Selten (1988) suggest that payoff dominance should be used instead of risk dominance. Since this however, a growing amount of support for the risk dominant equilibrium has emerged (Kandori, Mailath, and Rob, 1993; Young, 1993; Carlsson and Damme, 1993). Rather than suggesting that one equilibrium is played in a specific environment, this paper shows that there is hysteresis in coordination games, which means that different equilibria may be selected in a single environment depending on the history.

Another approach to better understanding the equilibrium selection problem is to run experiments with human subjects in the laboratory. The game commonly used to model these situations in the laboratory is the minimum-effort coordination game (or weak-link game). Experimental results suggest that coordination is more difficult with larger groups and higher costs of effort (Cooper, DeJong, Forsythe, and Ross, 1990; Van Huyck, Battalio, and Beil, 1990; Goeree and Holt, 2005). Building on these results, others have examined how changes in the environment can lead to higher levels of coordination. Some examples include adding communication (Cooper, DeJong, Forsythe, and Ross, 1992), competition between groups (Myung, 2011), and entrance fees (Cachon and Camerer, 1996). The results in this paper suggest that since there is hysteresis, changes in the environment only need to be temporary to result in an increase in the amount of coordination.

Systems exhibiting hysteresis are by no means rare. It is present in a wide variety of physical settings including magnetism and elasticity, but has also been observed in economics. Blanchard and Summers (1986) present a model in which the natural unemployment rate exhibits hysteresis in the presence of shocks. Employers make employment decisions in advance with the goal of maintaining steady employment in expectation. Employment shocks change these expectations and lead to more permanent changes in the natural unemployment rate. Baldwin (1988) shows that overvaluation of the dollar leads to hysteresis in United States import prices. Dixit (1989) examines entry of Japanese firms into the US market based on exchange rate fluctuations and finds that due to sunk costs, firm may remain in US even after the favorable exchange rate fluctuation has subsided. Nyberg (1997) examines an evolutionary model of honesty, and finds that once a society loses its honesty, hysteresis makes it difficult to reestablish. Finally, Göcke (2002) has a more detailed survey on some of the work that has been done examining hysteresis in economics. This paper focuses on coordination games, and finds that hysteresis occurs in these settings as well.

There have been some experimental papers that provide evidence of this type of hysteresis in coordination games. Weber (2006) examines the effect of changing group sizes on the group's ability to coordinate in a minimum-effort coordination game. He finds that coordination in large groups is possible if the group starts with a small number of subjects, and gradually increases to a size of 12 subjects per group. This is in contrast to groups that start with 12 subjects per group, which are never able to coordinate on high-effort levels. This suggests that there is hysteresis based on the group size, because the selected equilibrium for group size 12 depends on the history leading up to that game. Brandts and Cooper (2006) examine the effect of using payoff bonuses as a means of inducing cooperation in the minimum-effort coordination game. They find that adding bonuses helps bring groups from low effort levels to higher effort levels. In addition, they find that when the payoffs are decreased back to initial levels after the temporary bonuses, effort levels are higher than before the bonuses. This dependence on the history of the game is the type of behavior studied in this paper.

The paper proceeds as follows. The next section gives some intuition for the model, and what would cause hysteresis. Section 3 introduces the model. Next, Section 4 gives the theoretical results and experimental hypotheses. Section 5 presents the experimental design and the experimental results. Finally, Section 6 concludes.

2 Hysteresis Intuition

The main focus of this paper is determining when hysteresis occurs, and examining the implications. This section provides a brief example to give an idea of what an equilibrium correspondence with hysteresis looks like, and some of the implications. The example equilibrium correspondence is the s-shaped equilibrium correspondence displayed in Figure 1. This type of correspondence is referred to as double saddle-node bifurcation.[1] A double saddle-node is a bifurcation such that there is a unique solution in the limit of each direction, but three solutions inside some region in the middle as displayed in Figure 1.

If solutions tend to stay on the solution path (they do not jump between the equilibrium solutions as a parameter is perturbed), then the double saddle-node bifurcations leads to hysteresis. For example, consider the double saddle-node bifurcation in Figure 1. In this example, suppose that the parameter γ varies first from 0.2 to 0.8 (denoted by the red line), then it decreases from 0.8 back to 0.2 (denoted by the blue line). In this case, the system starts at $\gamma = 0.2$, where there is a unique equilibrium. As γ increases, the system remains on the top part of the s-shaped curve, until it reaches the saddle-node bifurcation at $\gamma = 0.58$ at which point the high equilibrium ceases to exist. For $\gamma > 0.58$, there is a unique equilibrium so the system jumps from the high equilibrium down to the low equilibrium (jump denoted by dotted line). The system remains in the low equilibrium as γ increases from 0.58 to 0.8. When γ decreases from 0.8, it remains on the low solution until $\gamma = 0.42$, at which point the low equilibrium ceases to exist, and again there is a unique equilibrium. This causes the system to jump back up from the low equilibrium to the high equilibrium. Therefore, for intermediate values, $\gamma \in (0.42, 0.58)$, the outcome depends on the starting position. When starting with $\gamma = 0.2$, the system goes to the equilibrium $\sigma = 0.94$ at $\gamma = 0.5$. When starting with $\gamma = 0.8$, the system goes to the equilibrium $\sigma = 0.058$ at $\gamma = 0.5$. This hysteresis is caused by the s-shaped equilibrium correspondence, and the assumption that the system traces along the equilibrium correspondence as the parameter changes.

Another implication of this double-saddle node bifurcation is that temporary changes in parameters may lead to large changes in outcomes. For example, consider the situation where $\gamma = 0.5$ and the equilibrium chosen is $\sigma = 0.058$. The system moves from the low σ equilibrium to the high

[1] A saddle-node bifurcation point is the point on the equilibrium correspondence where the number of equilibria is n to one side of the bifurcation point and $n + 2$ to the other side.

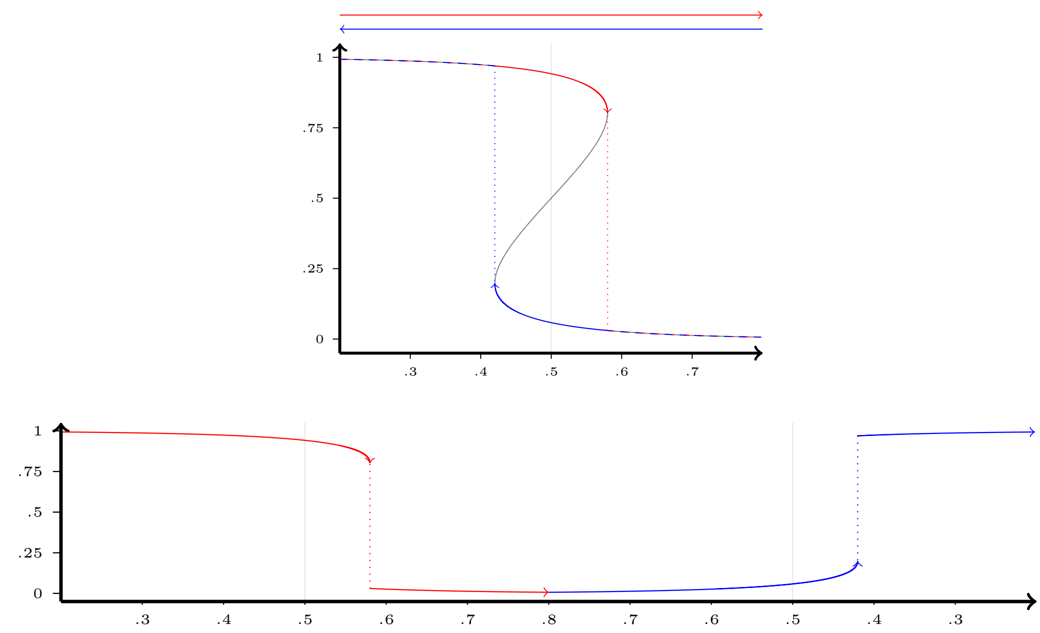


Figure 1: Example of how double saddle-node bifurcation leads to hysteresis.

σ equilibrium if γ is temporarily decreased and then changed back to $\gamma = 0.5$. This is true, because when γ is decreased to $\gamma = 0.2$, the low σ equilibrium ceases to exist, so the system automatically goes to the high σ equilibrium. However, since the solutions stay on path, when γ is increased back to $\gamma = 0.5$, the system remains at the high σ equilibrium. So a temporary change in the parameter can move the system from one equilibrium to another.

The next section introduces the model, and then examines situations where hysteresis occurs.

3 Model

Let $g(\gamma)$ be a game consisting of n players, $I = \{1, 2, \dots, n\}$. Each player has m pure actions, $S_i = \{s_i^1, s_i^2, \dots, s_i^m\}$. A joint-action profile is denoted by $\boldsymbol{s} = \{s_1, \dots, s_n\}$. Each player faces a payoff function $u_i(\boldsymbol{s}, \gamma)$ that depends on the parameter γ from parameter space Γ.

The set of mixed strategies is denoted by $\Sigma_i = \Delta^i$, which is the set of probability distributions over S_i. A mixed strategy is denoted by $\sigma_i \in \Sigma_i$, which is a mapping from S_i to Σ_i, where $\sigma_i(s_j)$ is the probability that player i plays pure-action s_j, and $\Sigma = \Sigma_1 \times \cdots \Sigma_n$ is the set of mixed strategy profiles. A joint mixed-strategy profile is denoted $\boldsymbol{\sigma} = \{\sigma_1, \dots, \sigma_n\}$. Player i's expected payoff for mixed-strategy profile $\boldsymbol{\sigma}$ is $u_i(\boldsymbol{\sigma},\gamma) = \sum_{\boldsymbol{s}\in S_1\times\cdots\times S_n} p(\boldsymbol{s})u_i(\boldsymbol{s},\gamma)$, where $p(\boldsymbol{s}) = \Pi_{i\in I}\sigma_i(s_i)$ is the probability of the pure-strategy profile $\boldsymbol{s}$ give mixed strategy profile $\boldsymbol{\sigma}$.

A joint strategy profile $\boldsymbol{\sigma}$ is an equilibrium of the game $g(\gamma)$ if the equilibrium function $f : \Sigma\times\Gamma\times\Lambda\to\mathbb{R}$, dependent on parameter $\lambda\in\Lambda$ satisfies $f(\boldsymbol{\sigma},\gamma,\lambda) = 0$. For example, f could be the logit quantal response equilibrium function,

$$f(\boldsymbol{\sigma},\gamma,\lambda) = \sum_{i=1}^{n}\sum_{j=1}^{m}\left|\frac{e^{\lambda u_i\left(s_j,\boldsymbol{\sigma}_{-\boldsymbol{i}},\gamma\right)}}{\sum_{k=1}^{m} e^{\lambda u_i\left(s_k,\boldsymbol{\sigma}_{-\boldsymbol{i}},\gamma\right)}} - \sigma_i(s_j)\right| = 0 \tag{1}$$

Given γ and λ, any joint mixed-strategy profile $\boldsymbol{\sigma}$ is an equilibrium if (1) is satisfied. The game $g(\gamma)$ has multiple equilibria for parameter λ if $f(\boldsymbol{\sigma},\gamma,\lambda) = 0$ for more than one joint mixed-strategy profile $\boldsymbol{\sigma}$. Let $\boldsymbol{\Sigma}^*(\gamma,\lambda) = \{\boldsymbol{\sigma}|f(\boldsymbol{\sigma},\gamma,\lambda) = 0\}$ be the set of equilibria of game $g(\gamma)$ according to equilibrium function f with parameter λ.

Definition 3.1 *The equilibrium correspondence $\boldsymbol{\Sigma}^*(\gamma,\lambda)$ varies continuously from $\underline{\gamma}$ to $\bar{\gamma}$ starting at $\underline{\boldsymbol{\sigma}}\in\boldsymbol{\Sigma}^*\left(\underline{\gamma},\lambda\right)$ if and only if for all $\varepsilon > 0$, there exists some $N\in\mathbb{N}$ such that for all $k = 0,\ldots,N-1$,*

$$\gamma_{k+1} - \gamma_k = \frac{\bar{\gamma}-\underline{\gamma}}{N} \Rightarrow \|\boldsymbol{\sigma}_{k+1} - \boldsymbol{\sigma}_k\| < \varepsilon$$

where $\gamma_0 = \underline{\gamma}, \gamma_N = \bar{\gamma}$ and $\boldsymbol{\sigma}_k\in\boldsymbol{\Sigma}^*(\gamma_k,\lambda)$. The endpoint of this continuous path is $\bar{\boldsymbol{\sigma}}\in\boldsymbol{\Sigma}^*(\bar{\gamma},\lambda)$.

The equilibrium correspondence varies continuously if you can trace the correspondence between the two parameter values while always moving in the direction from the first parameter value to the second parameter value. For example, in Figure 2, the equilibrium correspondence varies continuously from γ_1 to γ_2 starting at $\boldsymbol{\sigma}_2$. However, the equilibrium correspondence *does not* vary continuously from γ_1 to γ_2 starting at $\boldsymbol{\sigma}_1$.

Assumption #1: When faced with $g(\gamma)$, players will play one of the equilibria, call this $\boldsymbol{\sigma}(g(\gamma))\in\boldsymbol{\Sigma}^*(\gamma,\lambda)$.

Assumption #2: If game $g(\gamma)$ is played, and players play equilibrium $\boldsymbol{\sigma}(g(\gamma))$, then when game $g(\gamma')$ is played, if the equilibrium correspondence $\boldsymbol{\Sigma}^*(\gamma,\lambda)$ varies continuously from γ to γ' starting at $\boldsymbol{\sigma}(g(\gamma))$ with endpoint $\boldsymbol{\sigma}'$. Then when $g(\gamma')$ is played, players will play $\boldsymbol{\sigma}'$, that is $\boldsymbol{\sigma}(g(\gamma')) = \boldsymbol{\sigma}'\in\boldsymbol{\Sigma}^*(\gamma',\lambda)$.[2]

[2]Sheremeta, Cason, and Savikhin (2010) find cooperation spillover when moving from median-effort game to the

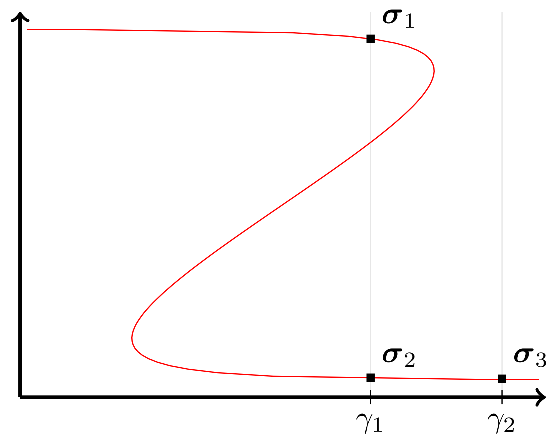


Figure 2: Example of continuous variation of a correspondence between two parameter values.

Definition 3.2 (Hysteresis) *The equilibrium correspondence $\boldsymbol{\Sigma}^*(\gamma, \lambda)$ exhibits hysteresis for equilibrium function f with parameter λ if there exists points $\gamma_1, \gamma_2, \gamma_3$ such that,*

1. *the correspondence varies continuously from γ_1 to γ_2 with starting point $\boldsymbol{\sigma}_1 \in \boldsymbol{\Sigma}^*(\gamma_1, \lambda)$ and endpoint $\boldsymbol{\sigma}_2 \in \boldsymbol{\Sigma}^*(\gamma_2, \lambda)$,*

2. *the correspondence varies continuously from γ_3 to γ_2 with starting point $\boldsymbol{\sigma}_3 \in \boldsymbol{\Sigma}^*(\gamma_3, \lambda)$ and endpoint $\boldsymbol{\sigma}'_2 \in \boldsymbol{\Sigma}^*(\gamma_2, \lambda)$, and*

3. *$\boldsymbol{\sigma}_2 \neq \boldsymbol{\sigma}'_2$.*

What types of games have this hysteresis property? The next section examines the minimum-effort coordination game, and shows that it exhibits hysteresis.

3.1 Minimum-Effort Coordination Game

A minimum-effort coordination game consists of n players, $I = \{1, \ldots, n\}$. Each player has two actions, they can either choose to exert high effort or low effort, $S_i = \{x_L, x_H\}$ for $x_L, x_H \in \mathbb{R}$ and $x_L < x_H$. The joint pure-action profile is denoted by $\boldsymbol{s} \in \{x_L, x_H\}^n$. Performing the high effort is more costly than performing the low effort. The benefit of the high effort is only received if every

minimum-effort game. This result suggests that there are behavioral spillover between games, which would provide some support for Assumption #2.

		$s_j = x_h$ for all $j \neq i$	$s_j = x_L$ for some $j \neq i$
Player i	x_H	$x_H\left(1-c\right)$	$x_L - cx_H$
	x_L	$x_L\left(1-c\right)$	$x_L\left(1-c\right)$

Figure 3: Minimum-Effort Coordination Game.

player plays the high effort action. If any player chooses the low effort action, then all players only receive the benefit from the low action. This yields payoffs,

$$u_i\left(\boldsymbol{s}\right) = \min_{j=1,\ldots,n} s_j - cs_i.$$

The normal form of the minimum-effort coordination game is displayed in Figure 3. For a given value of cost, $c \in \mathbb{R}$, the minimum-effort coordination game is denoted by $cg(c)$. The set of all minimum-effort coordination games is $\mathcal{CG} = \{cg(c)|c \in \mathbb{R}\}$.

3.2 Nash Equilibria

If $c > 1$, then the cost of exerting high effort outweighs the benefit, so action x_L strictly dominates x_H for all players. Therefore all players playing x_L is the pure-strategy Nash equilibrium when $c > 1$. Similarly, if $c < 0$, then the cost is negative, so the action x_H strictly dominates x_L. So, all players playing x_H is the unique Nash equilibrium. When $c \in [0,1]$, the game has two pure strategy Nash Equilibria: one where everyone plays the high effort x_H, and one where everyone plays the low effort x_L. There is also one symmetric mixed strategy equilibrium where all players play x_H with probability $c^{\frac{1}{N-1}}$, which is clearly increasing in N for $c \in (0,1)$.

For all values of $c \in (0,1)$, the equilibrium where all players play x_H with probability 1 is the *payoff dominant* equilibrium. For levels of c close to 1, the difference between the high effort equilibrium payoff and the low effort equilibrium payoff becomes small. However, there is a large loss possible if the high effort action is played, while there is no loss possible if the low effort is played. Therefore, when c is close to 1, the high effort action is risky.

3.3 Symmetric Quantal-Response Equilibria

This section studies properties of the symmetric quantal-response equilibria. Suppose that $\boldsymbol{\sigma}_H$ is the probability that a player plays the high effort action x_H, and $\boldsymbol{\sigma}_L = 1 - \boldsymbol{\sigma}_H$ is the probability that a player plays the low effort equilibrium. Using the logit quantal response function, a SQRE must satisfy the equation,

$$f(\boldsymbol{\sigma}_H, c, \lambda) = \frac{1}{1 + e^{\lambda[(x_h - x_L)(\boldsymbol{\sigma}_H{}^{N-1} - c)]}} - \boldsymbol{\sigma}_H = 0. \tag{2}$$

When $\lambda = 0$, there is always a unique SQRE, $\boldsymbol{\sigma}_H = 0.5$. The intuition for this is that when one player is playing randomly ($\lambda = 0$), then it is the best response for the other players to play randomly as well. Secondly, in the limit as $\lambda \to \infty$, there are always three solutions because the set of SQRE approaches the set of Nash equilibria as $\lambda \to \infty$, and there are always three symmetric Nash Equilibrium for games with $c \in (0, 1)$. To sum up, for low λ players are not very responsive to payoff, and so the only equilibrium is for everyone to play essentially randomly. However, when λ increases, then players are more sensitive to payoffs, and the set of equilibrium becomes closer to the set of Nash equilibria. The next section examines the properties of the SQRE correspondence as the cost parameter c is varied.

4 Results

This section determines properties of the logit SQRE correspondence for the minimum-effort coordination game. The main result finds conditions that need to be satisfied in order to get an s-shaped SQRE correspondence, which as has been shown leads to hysteresis. The results are summed up in the following proposition,

Proposition 4.1 *For every coordination game, $g(c) \in \mathcal{CG}$:*

1. *There exists a λ^* such that the logit SQRE correspondence, $\mathbf{\Sigma}^*(c, \lambda)$, exhibits hysteresis for all $\lambda > \lambda^*$, where,*
$$\lambda^* = \left(\frac{N}{N-1}\right)^N \frac{1}{x_h - x_l}.$$

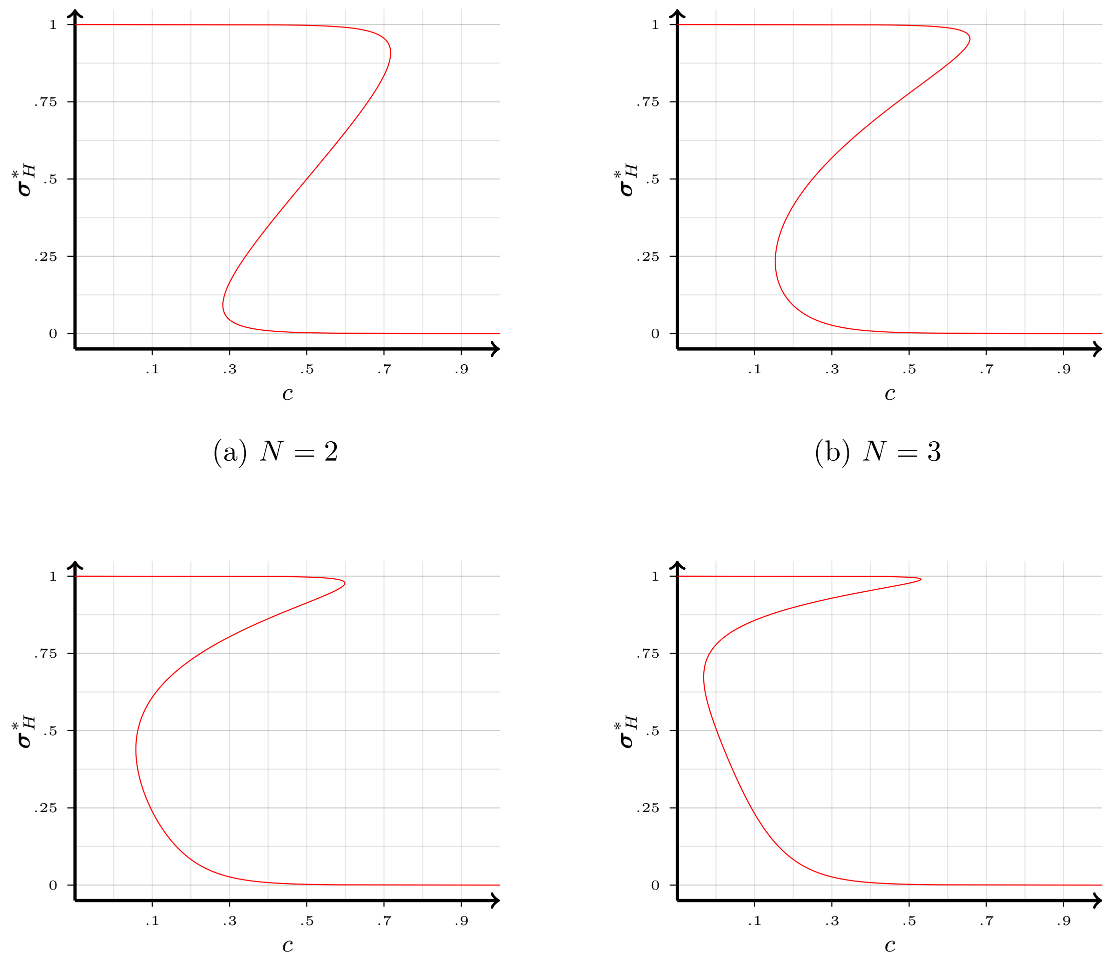
2. *The critical value λ^* is decreasing in N.*

3. *For $N = 2$, the saddle-node bifurcation points are given by,*
$$\sigma_H = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - \frac{4}{\lambda\left(x_h - x_L\right)}}.$$

4. *For $N > 2$, the saddle-node bifurcation points will not be symmetric around $\sigma_H = \frac{1}{2}$.*

This proposition states that for any coordination game of the given form, if the players have a high enough payoff responsiveness (sufficiently high λ), then the game should exhibit hysteresis. This means that given a game and an equilibrium, it is possible to vary one parameter slightly and then change it back, and the system could be at a completely different equilibrium. This can be very important if one of the equilibria is more desirable than the other and all that is required is a small perturbation of the system to go from the less desired to the more desired equilibrium. The second point in the proposition says that this critical value is decreasing as the size of the group gets larger. Assuming that the values of λ for the individuals are not dependent on group size, then this means that hystersis is more likely as the group size increases. The third part gives the analytical solution for the values of the saddle-node bifurcations for the $N = 2$ case. It is not possible to find the analytical solution for the $N > 2$ case, but with numerical analysis, it is clear that the c value of both saddle-nodes is decreasing as the group size gets larger (as shown in Figure 4). Also, for group size larger than two, the double saddle-node bifurcation is not symmetric, meaning that the bifurcation points are not equidistant from 0.5. However, in the $N = 2$ case, the double saddle-node bifurcation is symmetric.

The proof of this is given in the appendix. The proof of this proposition involves analyzing certain properties of the equilibrium correspondence. The double saddle-node bifurcation is not a function, so it is difficult to analyze. However, it is possible to solve for the equilibrium value of c as a function of σ_H, $c^*(\sigma_H)$, which is a function and therefore easier to work with. The equilibrium



(a) $N = 2$ (b) $N = 3$

(a) $N = 5$ (b) $N = 10$

Figure 4: QRE correspondences as c is varied for different value of N and $\lambda = 4$.

correspondence $\mathbf{\Sigma}^*(c, \lambda)$ has a double saddle-node bifurcation if $c^*(\sigma_H) \to \infty$ as $\sigma_H \to 1$, $c^*(\sigma_H) \to -\infty$ as $\sigma_H \to 0$, and $c^{*\prime}(\sigma_H) < 0$ for some value of $\sigma_H \in (0, 1)$. This process ensures existence of a double saddle-node bifurcation for sufficiently large values of λ. The rest of the proposition is obtained from comparative statics that are detailed in the appendix.

This setup yields some testable implications, the most important being that the equilibrium correspondence of the minimum-effort coordination game exhibits hysteresis. To test this, it is necessary to run an experiment with multiple games, where the games are varied in an organized manner to determine whether the experimental outcomes exhibit hysteresis or not. Based on the theoretical results, the experiment is used to test the following three hypotheses:

Hypothesis 1: In the minimum effort coordination game, there should be significantly higher effort levels at $c = 0.5$ in groups that start with a low cost that gradually increases to $c = 0.5$ than in groups that start with a high cost that gradually decreases to $c = 0.5$.

Hypothesis 2: If the cost is varied from low to high to low, then for an intermediate cost level c^*, there should be significantly **higher effort levels** the first time the group faces c^* (going from low to high) than the second time the group faces c^* (going from high to low).

Hypothesis 3: If the cost is varied from high to low to high, then for an intermediate cost level c^*, there should be significantly **lower effort levels** the first time the group faces c^* (going from high to low) than the second time the group faces c^* (going from low to high).

These hypotheses predict that there is hysteresis in the minimum-effort coordination games. The next section details the experimental methods and results.

5 Experiments

5.1 Experimental Design

The subjects were drawn from a pool of undergraduate students from Purdue University that were signed up to participate in experiments at the Vernon Smith Experimental Economics Laboratory. Upon entering the lab, the subjects were randomly assigned to a computer and given a handout containing the instructions. After all subjects had been seated, the instructions were read aloud.

After the instructions, each subject completed a quiz to make sure that they understood the format of the game (see supplementary material for instructions and quiz). The experiment did not start until all subjects had correctly answered all of the questions on the quiz. The experiment was programmed and conducted with the software z-Tree (Fischbacher, 2007) (For a screenshot of the experimental interface see supplementary material). During the experiments, all payoffs were displayed in US dollars. Subjects' final payoff was the sum of earnings from 3 randomly selected rounds. After the experiment, subjects were anonymously given their payment in cash. The average payoff was $12.50 and each session took about 45 minutes to complete including instructions.

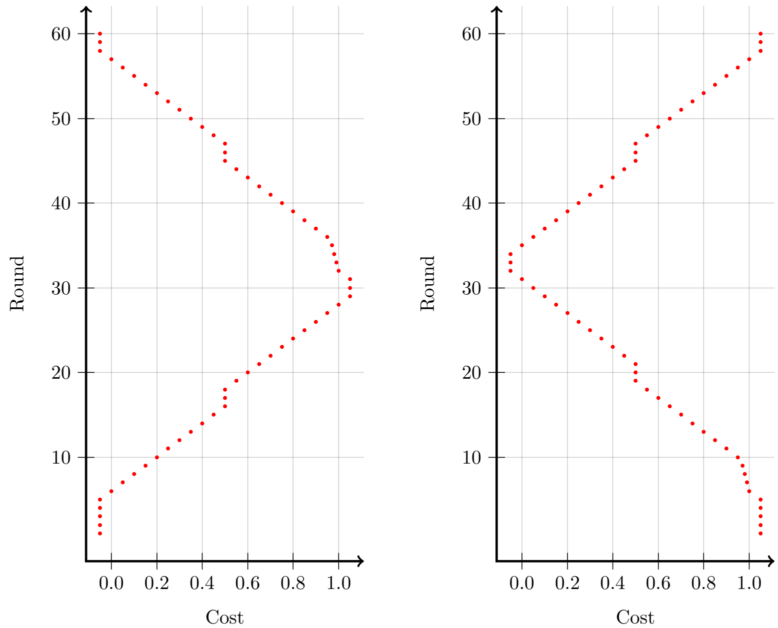
The experiment was divided into six sessions, each of which consisted of 60 rounds. At the beginning of the session, subjects were divided into groups of four, and remained in the same group for all 60 rounds. In each round, the subjects played a minimum-effort coordination game in which they were asked to choose one of two options, high effort (labeled X) or low effort (labeled Y). Based on the choices of all members of the group, if everyone chose high effort then the group choice was high effort (labeled X), and if anyone chose low effort then the group choice was low effort (labeled Y). Their payoffs were displayed in a table similar to that in Figure 3, with $x_H = 6$, $x_L = 1.5$, and varying cost parameters $c \in [-0.05, 1.05]$. In the table, all payoffs were multiplied out, so the subjects just saw a single number in each box. After all subjects had made their choice they got to see their payoff for the round, and their group choice.[3] The players also were asked to record their choices and payoffs from each round on a record sheet.

The experiment consisted of two types of sessions. One in which cost was varied from low to high to low (LHL), and another where cost was varied from high to low to high (HLH). LHL and HLH each had a total of 64 subjects divided into 16 groups. In the LHL sessions, the cost started at $c = -0.05$, gradually increased to $c = 1.05$, and then decreased back to $c = -0.05$. In the HLH sessions, the cost started at $c = 1.05$, gradually decreased to $c = -0.05$, and then increased back to $c = 1.05$. Figure 5 shows the full progression of the costs for each type of session.

5.2 Experimental Results

Table 1 displays effort rates at $c = 0.5$ for the different directions (whether cost is increasing to $c = 0.5$ or decreasing to $c = 0.5$) for both the LHL and HLH sessions.

[3] Note that the group choice is less information than seeing the choice of each member of the group.



(a) LHL Sessions (b) HLH Sessions

Figure 5: Progression of costs in LHL and HLH sessions.

5.2.1 Hypothesis 1

Hypothesis 1 predicted that there would be significantly more high-effort at $c = 0.5$ in groups that started at a low cost and gradually increased to $c = 0.5$ than in groups that started at a high cost and gradually decreased to $c = 0.5$. In other words, different groups have different outcomes depending on the starting point. In LHL, the groups started with a cost of $c = -0.05$ and then cost gradually increased until it reached $c = 0.5$ in rounds 16-18. In HLH, the groups started with a cost of $c = 1.05$ and then cost gradually decreased until it reached $c = 0.5$ in rounds 19-21. Table 1 shows that when players start with a low cost, 88% (169/192) of individuals play high effort, which leads to 85% (41/48) of groups attaining the high effort when $c = 0.5$ in rounds 16-18 of the LHL treatment. Alternatively when players start with a high cost, only 35% (67/197) of individuals play high effort, which leads to 13% (6/48) of groups attaining the high effort when $c = 0.5$ in rounds 19-21 of the HLH treatment. So when the group started at a low cost, and increased to $c = 0.5$, they attained the high effort 85% as compared to only 13% when the group started at a high cost and decreased to $c = 0.5$. This difference is significant using a one-tailed Wilcoxon Rank-Sum Test

Table 1: Rates of high effort of individuals and groups for each session.

Session	Rounds	Direction	Cost	% Indv. = High	% Group = High
LHL	16-18	Increasing	0.5	88% (169/192)	85% (41/48)
	45-47	Decreasing	0.5	71% (136/192)	56% (27/48)
HLH	19-21	Decreasing	0.5	35% (67/192)	13% (6/48))
	45-47	Increasing	0.5	72% (138/192)	69% (33/48)

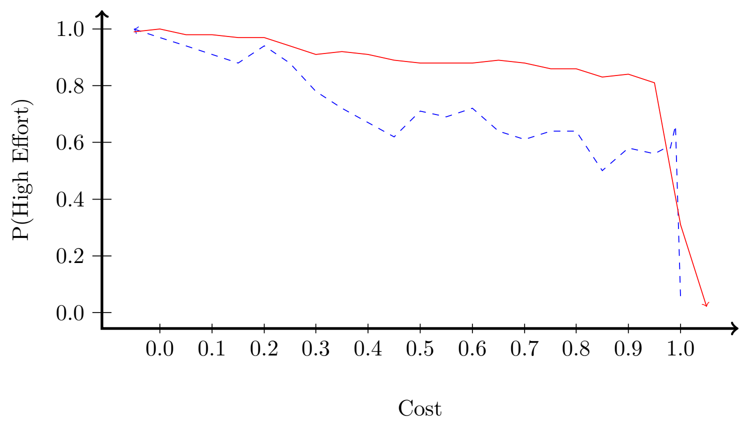
with normal approximation ($p = 8.5 \times 10^{-5}$).[4] This provides strong support for hypothesis 1, and suggests that there is hysteresis in the minimum-effort coordination game.

5.2.2 Hypothesis 2

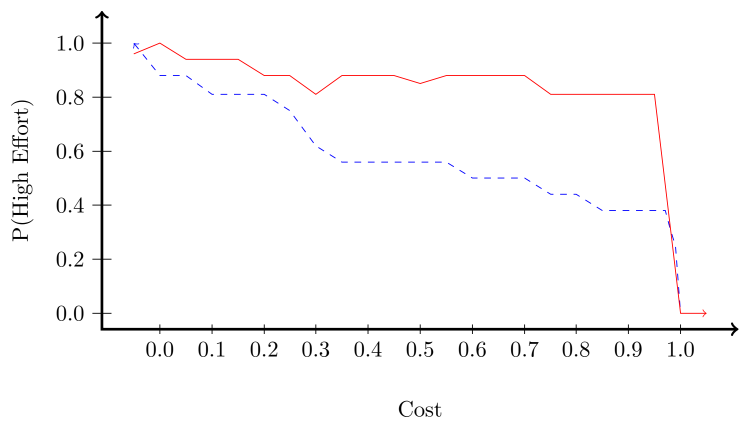
While Hypothesis 1 predicted that different groups will have different outcomes at $c = 0.5$ depending on the starting point, Hypothesis 2 and Hypothesis 3 go even further by predicting that the same group has different outcomes at $c = 0.5$ depending on whether the cost is increasing or decreasing. Hypothesis 2 predicted that in LHL, there would be significantly more high-effort when the subjects play the minimum-effort coordination game at $c = 0.5$ when the cost is changing from low to high than when the cost in going from high to low. In LHL the cost starts at $c = -0.05$ and gradually increases to $c = 0.5$ in rounds 16-18. Then the cost increases more to $c = 1.05$ before it starts to decrease again, and reaches $c = 0.5$ again in rounds 45-47. Table 1 shows details of the level of high effort at the individual and group level in the LHL sessions. As Hypothesis 2 predicted, a one-tailed Wilcoxon Rank-Sum Test with normal approximation ($p = 0.045$) confirms that there is significantly more high effort when the cost is increasing than when it is decreasing at $c = 0.5$.

Figure 6 shows the full progression of the level of high effort for both individuals and groups. Figure 6(a) shows the individual choices. One striking observation from this plot is that when the cost was $c = 0.99$, 66% (42/64) of subjects played high effort in the game. In this game, subjects could guarantee a payoff of \$0.02 by playing low effort. If the subjects played high effort then they

[4] To test the signifigance of the difference in efforts between the two sets of groups, a Wilcoxon rank-sum test is used. Each group plays at $c = 0.5$ for three periods, and receives a score of 0-3 depending on how many times they attained the high effort in those three periods. Rank-sums are then determined for these sets, and a normal approximation is used.



(a) High Effort Individuals



(b) High Effort Groups

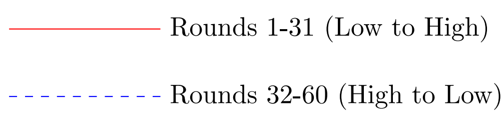


Figure 6: Progression of high effort in LHL for individuals and groups.

would get \$0.06 if everyone else played high effort and -\$4.44 if one other person choose low effort.

5.2.3 Hypothesis 3

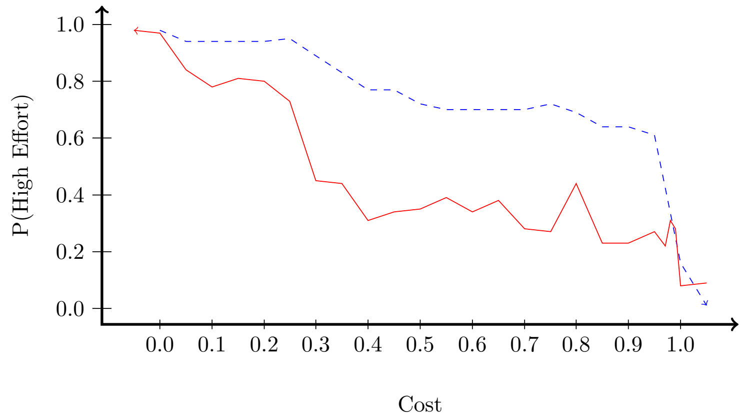
Hypothesis 3 predicted that in HLH, subjects would exert significantly less high effort the first time they play at $c = 0.5$ (as cost is decreasing) compared to the second time they play at $c = 0.5$ (as cost is increasing). Table 1 shows that 13% of groups attain the high effort as the cost is decreasing while 69% attain the high effort as the cost is increasing. The difference in levels of high effort groups is significant using a one-tailed Wilcoxon Rank-Sum Test with normal approximation ($p = 1.6 \times 10^{-3}$). This shows that a temporary decrease in the cost can have a large impact on equilibrium selection. This temporary decrease in the cost helped increase the percentage of high effort groups from 13% to 69%. These results strongly support Hypotheses 3. Figure 7 shows the full progression of both group and individual choices in the HLH sessions.

6 Conclusion

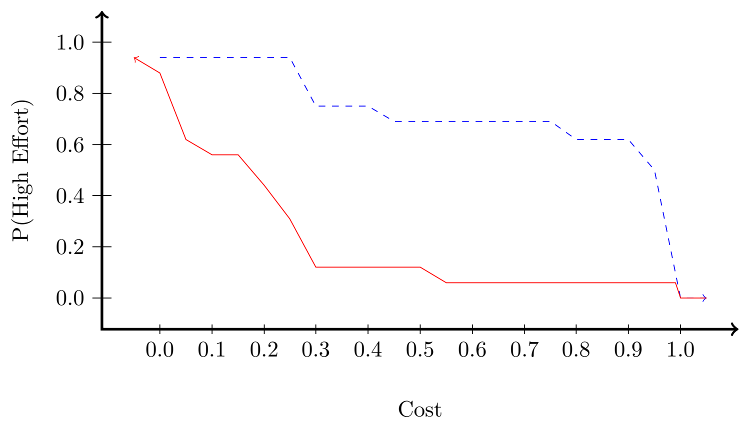
This paper started with the idea that hysteresis, a property that is common in wide range of physical settings, is also present in an economic system. Proposition 4.1 shows that the minimum-effort coordination game exhibits hysteresis as long as several assumptions are satisfied. Based on this theoretical result, some experimental hypotheses were developed and then tested in the lab using human subjects. The experimental results presented in Section 5 provide support for the theoretical results, suggesting that hysteresis occurs in the minimum-effort coordination game.

Probably the most important implication of this hysteresis is on equilibrium selection. The experimental results showed that in a minimum-effort coordination game with cost $c = 0.5$, 13% of groups played high effort when the cost was decreasing, as opposed to 72% of groups when the cost was increasing. This suggests that the same group of people can behave one way in a certain game, but then behave completely differently in the same game after a parameter has been temporarily changed. With regard to equilibrium selection, the suggests that we may be able to move from one equilibrium to a better equilibrium by just temporarily changing certain parameters.

Another interesting implication of the s-shaped curve that was not studied in this paper is the fact that it can lead to oscillations or cycles in the system. In the experiments presented here,



(a) High Effort Individuals



(b) High Effort Groups

Rounds 1-31 (Low to High)

Rounds 32-60 (High to Low)

Figure 7: Progression of high effort in HLH for individuals and groups.

the cost was changed exogenously. However, in many situations, the parameters may be changing endogenously. For example, if coordination on the high effort pushes the cost upward while the groups that exert low effort have decreasing costs, then this would lead to a cycle. The effect of this cycling on equilibrium selection provides some interesting questions for future work.

Given the implications of this hysteresis it is important as determine how likely this hysteresis is in a framework more general the minimum-effort coordination game. Though this topic is not discussed in this paper, it is hypothesized that the s-shaped curve is not a knife-edge case, but is likely to occur in a large number of settings as well. Better understanding when this s-shaped curve occurs may help provide better understanding of the equilibrium selection process in more general settings. Further examination of the genericity of this hysteresis is saved for future work.

Finally, the model presented in this paper is quite simple, and does not factor in a lot of things that are likely going on as the parameters change in these games. However, even this simple model is able to provide clean predictions about what we see in the experimental lab, so for this paper it is sufficient. Another interesting avenue for future work would be to try to develop a more sophisticated model of learning in close games, and see if this provides any additional insights about phenomena studied in this paper.

A Proofs

Proof of Proposition 4.1: First, calculate the symmetric quantal response equilibrium of the game. Suppose that all players play x_H with probability σ_H and x_L with probability σ_L. So the probabilities are,

$$P\left(\text{All others play H}\right) = \sigma_H^{N-1}, \text{ and}$$

$$P\left(\text{At least one other L}\right) = 1 - \sigma_H^{N-1}.$$

The payoffs are as follows,

$$\begin{aligned} u_i\left(x_L, \sigma_{-i}\right) &= \left(1-c\right) x_L \\ u_i\left(x_H, \sigma_{-i}\right) &= x_L(1-\sigma_H^{N-1}) + x_H \sigma_H^{N-1} - c x_H \\ &= x_L - c x_H + \sigma_H^{N-1}\left(x_H - x_L\right). \end{aligned}$$

Therefore, the symmetric logit quantal response equilibrium must satisfy the following equations,

$$\begin{aligned}
\sigma_H &= \frac{e^{\lambda u_i(x_H,\sigma_{-i})}}{e^{\lambda u_i(x_L,\sigma_{-i})}+e^{\lambda u_i(x_H,\sigma_{-i})}} \\
&= \frac{1}{1+e^{\lambda[u_i(x_L,\sigma_{-i})-u_i(x_H,\sigma_{-i})]}} \\
&= \frac{1}{1+e^{\lambda(x_H-x_L)\left(c-\sigma_H^{N-1}\right)}}. \qquad (3)
\end{aligned}$$

In order to show that hysteresis is possible, it is necessary to show that the bifurcation correspondence, $\mathbf{\Sigma}^*(c,\lambda)$, has the double saddle-node bifurcation. To do this, we find $c^*(\sigma_H)$, which is a function. Next, show that $\lim_{\sigma_H\to 0} c^*(\sigma_H)\to -\infty$, $\lim_{\sigma_H\to 1} c^*(\sigma_H)\to\infty$, and $c^{*\prime}(\sigma_H)<0$ for some $\sigma_H\in(0,1)$. If these conditions hold then, the bifurcation correspondence, $\mathbf{\Sigma}^*(c,\lambda)$, has a double saddle-node bifurcation and look like an s-shaped curve. Rearrange Equation (3) to get,

$$c^*(\sigma_H) = \sigma_H^{N-1} + \frac{\ln\frac{1-\sigma_H}{\sigma_H}}{\lambda\left(x_H-x_L\right)}. \qquad (4)$$

Which has a unique value for c^* for each value of σ_H. From this, notice that,

$$\lim_{\sigma_H\to 0} c^*(\sigma_H) \to \frac{\ln\infty}{\lambda\left(x_H-x_L\right)} = \infty$$

and,

$$\lim_{\sigma_H\to 1} c^*(\sigma_H) \to 1 + \frac{\ln 0}{\lambda\left(x_H-x_L\right)} = \infty.$$

Finally,

$$\frac{\partial c^*}{\partial\sigma_H}(\sigma_H) = (N-1)\,\sigma_H^{N-2} - \frac{1}{\lambda\left(x_H-x_L\right)}\left(\frac{1}{\sigma_H\left(1-\sigma_H\right)}\right). \qquad (5)$$

Therefore,

$$\frac{\partial c^*}{\partial\sigma_H}(\sigma_H) > 0 \iff \lambda > \frac{1}{(N-1)\,p\,(1-p)^{N-1}\left(x_H-x_L\right)}.$$

In order to get the s-shaped curve, this needs to hold for some $\sigma_H\in(0,1)$. Since the right side of the above equation is minimized when $\sigma_H = {}^1/_N$, so see that,

$$\begin{aligned}\frac{1}{(N-1)\,\sigma_H\,(1-\sigma_H)^{N-1}\,(x_H - x_L)} &\geq \frac{1}{(N-1)^1/_N(1-{}^1/_N)^{N-1}(x_H-x_L)} \\ &\geq \frac{1}{\frac{N-1}{N}^N(x_H-x_L)} \\ &= \left(\frac{N}{N-1}\right)^N \frac{1}{x_H-x_L}.\end{aligned}$$

Therefore, if

$$\lambda^* = \left(\frac{N}{N-1}\right)^N \frac{1}{x_H - x_L},$$

then for all $\lambda \geq \lambda^*$, the bifurcation correspondence $\boldsymbol{\Sigma}^*(c,\lambda)(c)$ has the desired s-shaped form. Also, note that,

$$\left(\frac{N}{N-1}\right)^N \geq \left(\frac{N+1}{N}\right)^{N+1} \quad \text{for all } N \geq 2.$$

This holds by the Bernoulli Inequality[5]. Therefore, λ^* is decreasing in N. This means that as the group size increases, holding everything else constant, the s-shaped curve is more likely.

Finally for any fixed value of λ, the saddle-node bifurcation points of the s-shaped curve are at the two points where,

$$\frac{\partial c}{\partial \sigma_H}(\sigma_H) = 0.$$

Then set (5) to zero, and rearrange to get,

$$(1-\sigma_H)\,\sigma_H^{N-1} - \frac{1}{\lambda\,(x_H - x_L)\,(N-1)} = 0. \tag{6}$$

[5] The Bernoulli Inequality says that for $1 \geq \alpha > 0$ and $\delta \geq -1$,

$$(1+\delta)^\alpha \leq 1 + \alpha\delta.$$

Set $\alpha = {}^N/_{N+1}$ and $\delta = -\frac{1}{N}$, then the inequality shows that,

$$\left(1 - \frac{1}{N}\right)^{N/N+1} \leq 1 - \frac{N}{N+1} \times \frac{1}{N} = \frac{N}{N+1}.$$

Taking the reciprocal,

$$\left(\frac{N}{N-1}\right)^{N/N+1} \geq \frac{N+1}{N}.$$

Or equivalently,

$$\left(\frac{N}{N-1}\right)^N \geq \left(\frac{N+1}{N}\right)^{N+1}.$$

An explicit solution for this equation in not tractable unless $N = 2$. In the $N = 2$ case, solving this gives the solution

$$\begin{aligned}\sigma_H &= \frac{\lambda\left(x_H - x_L\right) \pm \sqrt{\lambda^2\left(x_H - x_L\right)^2 - 4\lambda\left(x_H - x_L\right)}}{2\lambda\left(x_H - x_L\right)} \\ &= \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - \frac{4}{\lambda\left(x_H - x_L\right)}}.\end{aligned}$$

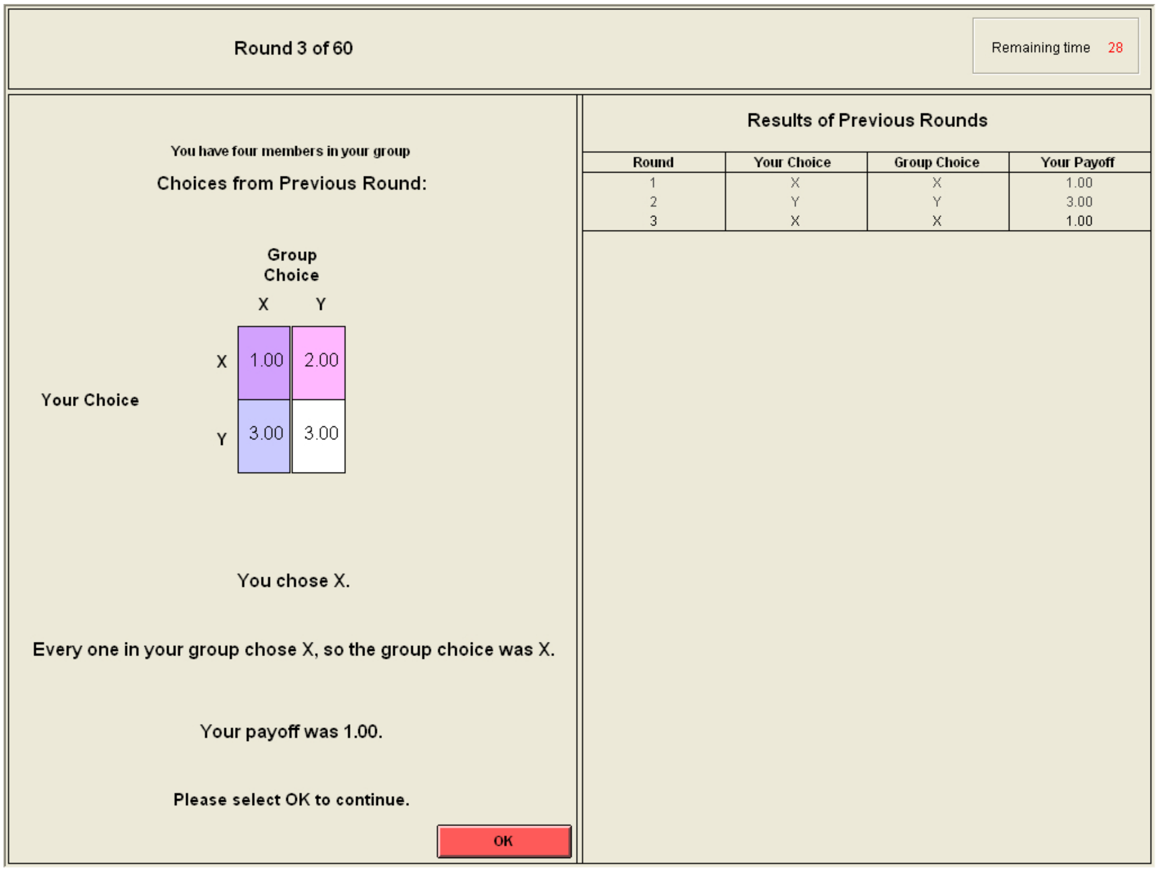
Also see that if $\lambda \geq \lambda^* = \frac{4}{x_H - x_L}$, then

$$1 \geq \frac{4}{\lambda\left(x_H - x_L\right)}.$$

So if $\lambda \geq \lambda^*$, then the two roots are always real, and if $\lambda < \lambda^*$, then there are no real roots, which is what we would expect. These two saddle-node bifurcation points are symmetric around $\sigma_H = \frac{1}{2}$ for the $N = 2$ case. However, for the $N > 2$ case, we would not expect to see this symmetry due to the form of (6).

B Experimental Materials

B.1 Screenshot



B.2 Instructions and Quiz

Experiment Overview

You are about to participate in an experiment in the economics of decision-making. If you listen carefully and make good decisions, you could earn a considerable amount of money that will be paid to you in cash at the end of the experiment.

Please do not talk or communicate with other participants. Feel free to ask questions by raising your hand or signaling to the experimenter.

We will first go over the instructions, then give a quiz to make sure everyone understands the instructions. After the quiz, the experiment will begin.

Experiment Details

This experiment consists of 60 rounds. At the beginning of the experiment, everyone is randomly assigned into groups of 4 people. Your group consists of you and 3 other members. You will remain in the same group for all 60 rounds.

Specific Instructions for Each Round

Each round you will be asked to choose one of two options, X and Y. Based on your choice and the choice of the other members of your group, a group choice will be determined.

Group Choice

The group choice is determined as follows:

- If **EVERYONE** in your group chooses X, then the group choice is X.
- if **ANYONE** in your group (including yourself) chooses Y, then the group choice is Y.
- Examples,
  - Individual Choices (X,X,X,Y), then the group choice is Y
  - Individual Choices (Y,Y,Y,X), then the group choice is Y
  - Individual Choices (Y,Y,Y,Y), then the group choice is Y
  - Individual Choices (X,X,X,X), then the group choice is X

After each round you will see the "Group Choice" but you will not be able to see the individual choices of all members of your group.

Payoffs

All payoffs in the experiment are displayed in $US. At the end of the experiment, you will be paid in cash. Your payoff at the end of the experiment will be the sum of the payoffs for **3 randomly selected rounds** from the 60 total rounds in the experiment.

When making your choice, your payoffs will be displayed in a table like the one displayed below.

Your Choice		Group Choice X	Group Choice Y
	X	1.00	2.00
	Y	3.00	3.00

To make your choice, click the button marked either X or Y. After you have made your choice, the corresponding row will become highlighted.

Your Choice		Group Choice X	Group Choice Y
	X	1.00	2.00
	Y	3.00	3.00

In the above example, the person choose X. Once you have made your choice, you will wait for the other members of your group to make their choice. Once everyone in your group has made their choice, the group choice column will be highlighted.

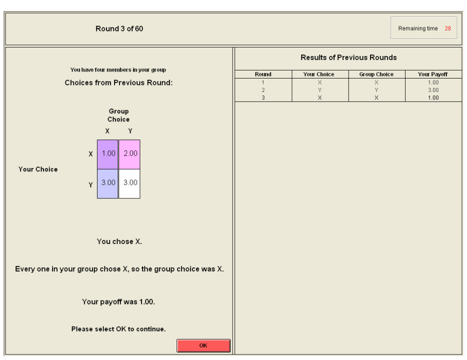
Your Choice		Group Choice X	Group Choice Y
	X	1.00	2.00
	Y	3.00	3.00

Your payoff will be the square that corresponds to your choice and the group choice. In the above example, the payoff is $2, because "Your Choice" was X and the "Group Choice" was Y.

IMPORTANT:

- Each round the payoffs may change, so be sure to look at the payoffs each round.
- The payoffs may be negative. If you earn negative total profits it will be taken from your show-up fee.
- After each round, please record "Your Choice," your "Group Choice" and your "Payoff" for the round on your record sheet.

Screenshot



- The round number is in the top left.
- The choice table is on the left half of the screen.
  - "Your Choice" is represented by the rows.
  - The "Group Choice" is represented by the columns.
- The history table is on the right half of the screen. This tells the "Round", "Your Choice", the "Group Choice", and "Your Payoff" for each round.
- The summary and status are is on the bottom left. This area will tell you:
  - When you are waiting for other subjects to make their choice
  - After the round is over, what your payoff was in the previous round.

Quiz

To make sure everyone understands the instructions, please answer the following questions.

1. How many rounds are there in the experiment? ______________
2. How many people are in your group including yourself? ______________
3. Your payoff is determined by the sum of how many randomly selected rounds?______________
4. What is your payoff in the following tables?

Your Choice		Group Choice X	Group Choice Y
	X	1.00	2.00
	Y	3.00	3.00

Your Payoff: ______________

Your Choice		Group Choice X	Group Choice Y
	X	1.00	2.00
	Y	3.00	3.00

Your Payoff: ______________

Your Choice		Group Choice X	Group Choice Y
	X	1.00	2.00
	Y	3.00	3.00

Your Payoff: ______________

5. What is the group choice in the following situations?
   - Everyone in your group chooses X. Group Choice: ______________
   - Everyone in your group chooses Y. Group Choice: ______________
   - Three people choose Y and one person chooses X. Group Choice: ______________
   - You choose Y and everyone else chooses X. Group Choice: ______________

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