

# Impact of Risk Specification on Valuation of Multi-dimensional Projects

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## Abstract

Motivated by business situations in a number of domains, we show, using laboratory experiments, that subjective valuations of multi-dimensional risky projects are highly sensitive to the format of risk specifications. We considered a project that has two dimensions, each with an associated risk. A project would be considered a success when favorable outcomes occur on both dimensions. Participants were provided with the probabilities of success for each dimension in two different specifications. In the simple risk specification, each probability was specified as a point estimate. In the compound risk specification, each probability was specified as a two-point distribution. Structural analysis of the data shows that the widely reported overestimation of conjunctive probabilities under simple risk specification is reduced by a large margin under a compound risk specification. We also show that this mitigation occurs because individuals make valuation judgments as if the reduced probability of success is lower than normative probability. Effectively, we show that the two biases in processing conjunctive events and compound risk specifications act in opposite directions, and as outcome, judgmental valuations for a two-dimensional project are closer to the true value with compound risk specification (versus a simple risk specification). These results have an important implication for communicating risk: substituting compound risk specifications with simple risk specifications can lead to a systematic deterioration in the quality of project valuations that involve multiple risks.

**Keywords:** Risk Specifications, Multiple Risks, Behavioral Biases

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# 1 Introduction

Businesses have progressively become more complex, and often new projects must succeed simultaneously on multiple dimensions. For example, new technologies routinely involve innovation on multiple dimensions that must be simultaneously successful; supply chains must be cheap and robust at the same time; construction projects must finish both on time and within budget to be considered a success. In an effort to better manage this multi-dimensional complexity, several firms have started to quantitatively specify various risks. HP (Ward et al., 2010), Bayer (Stonebraker, 2002), Intel (Wu et al., 2010; Kempf et al., 2013), among other firms, now require domain experts involved in various projects to specify risk information in the form of probabilities of specific outcomes. This risk information is then communicated to business managers, who use this information to compare various alternatives and make decisions. The quality of such decisions depends on managers correctly processing risk specifications communicated to them. However, there is scant literature on how managers behaviorally process risk information for multiple risks and whether their behavioral biases have any systematic effect on their decisions.

In this paper, we investigate how differences in the format in which risk information is communicated to managers affect their valuation of multi-dimensional projects. Specifically, we consider a two-dimensional project that is deemed a success only if the development effort is successful on both dimensions. Each dimension has an associated risk. Domain-experts have compiled the risk information for both risks, and they are considering two possible formats of this information – or risk specifications – to communicate to a decision maker. In *compound* risk specification probabilities are not fixed but have a distribution. In *simple* risk specification the probabilities are reduced to fixed values. Both risk specifications exist commonly in situations involving decision making under risk. We explore whether these risk specifications, even when they are normatively equivalent, lead to systematic differences in judgmental valuations of a project. Our focus on this issue is motivated by several examples from industry; we next discuss some of these examples.

## 1.1 Motivation: Business Situations Involving Multi-dimensional Compound Risks

We first discuss a specific business situation that motivate this research; and then discuss the widespread presence of such situations in industry. In 2007, IBM was planning to launch a server system that would be based on a new microprocessor chip, P7. There were two sources of risks in the development process: the processor and the heat sink.<sup>1</sup> The processor’s manufacturing yield had to be high enough for the system to be profitable. The initial efforts to manufacture the processor had suggested that several circuits were difficult to produce on a mass scale and needed to be modified. The heat sink also needed a major overhaul. The heat sink of the previous generation system would not dissipate the higher amounts of heat released by the new processor. IBM needed to cross *both* of these technological hurdles before it could launch the new system.

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<sup>1</sup>Computer processors dissipate heat in large amounts. This heat can raise the temperature of computer systems to the extent of melting crucial parts. To prevent this, a heat sink is attached at the back of the processor. The heat sink dissipates the heat produced by the processor to the external environment via a fan, among other parts.

The firm held regular meetings to discuss a potential launch date for the new system. During these meetings, the two risks were discussed in detail. The team-leads of the production team (responsible for processor-yield) and the design team (responsible for the new heat sink) gave their probability estimates that their individual tasks would be complete by the middle of that year. The project manager needed to make the launch date decision after taking both of these probability estimates into consideration. Initially, the team-leads provided a simple risk specification: they stated a 50% and a 30% probability (the actual numbers are modified for privacy purposes), respectively, that the development issues with the processor and the heat sink would be resolved successfully by the middle of that year. However, during deeper discussions both team-leads provided details that led to compound risk specifications. The design team-lead stated that he did not have prior data for developing similar heat sinks that were directly relevant to the current situation. His subjective estimate of 30% was his best estimate, but he added that his estimates could have some noise on a case by case basis. For the current effort he believed that the actual probability was 30% “plus-minus 10%”. The production team-lead stated that she faced a two step uncertainty — whether new circuits will be complex or simple, and if the yield will be high for the new circuits. The firm had prior experience in circuit-redesigns, and had maintained the data for such efforts. She stated that these data suggested that the probability that the circuits would turn out to be complex was 50%. Further if the new circuits were complex (simple) there was a 25% (75%) probability that the yield would be high. So effectively, the probability of success was either 25% or 75% with equal chances. We seek to investigate whether such a change in risk specification on multiple dimensions leads to systematic changes in project valuation.

### 1.1.1 Prevalence of Multi-dimensional Risks

Business situations that involve multiple risks, like the one at IBM, are common in industry. Krishnan and Bhattacharya (2002) discuss managerial decisions for the development of new laptops at Dell under two risks (market risk and battery development risk), and note that (p. 313) “[w]hile newer advanced technologies may offer improved performance, they also make the product development process more risky and challenging” and that “. . . the problem of technology selection and commitment under uncertainty . . . (is) a major challenge to firms in turbulent environments.” Other articles that investigate multi-dimensional technologies include Chhajed and Kim (2005); Matsubayashi (2008); Takagoshi and Matsubayashi (2013). Multi-dimensional risks widely exist in supply chain environments with multiple substitutable products each with a demand risk (e.g. Nagarajan and Rajagopalan, 2008); in production planning environments with supply and demand risks (e.g. Gerchak et al., 1994); in project management with uncertainty in cost and completion-time (e.g. Chapman and Ward, 1996); and in agribusiness where both the quality and quantity of produce may be uncertain (e.g. Ziggers and Trienekens, 1999). Our focus on the role of risk specifications in managerial judgments in multi-risk environments is relevant to these contexts, among others.

### 1.1.2 Operational Causes of Compound Risk Specifications

Compound risks are naturally present in businesses, for two reasons. The first reason is applicable to business situations in which prior data is not available for estimating probabilities of outcomes, and a firm relies on domain experts to obtain probability judgments. Experts' probability judgements are subject to errors (Clemen and Ulu, 2008). Cooke (1991) discusses a calibration exercise to quantify the uncertainty in the probability values provided by domain experts. This calibration leads to a two-tiered uncertainty — the point probability provided by the expert for an outcome, and given his probability judgment a distribution over the true probability — and the accompanying compound risk specification. For example, in IBM's case the design team-lead specified that his best estimate for the probability of success was 30% and, based on his experience with how well his estimates have performed in the past, this point estimate was subject to uniform uncertainty of 10% in either direction. Expert calibration has been used in a number of articles to rank order experts based on the errors documented during calibration and to combine their assessments (e.g. Merrick, 2008; Budescu and Chen, 2014; Turner et al., 2014). But an important question — how does the form of risk specifications (simple or compound) for multiple risks provided by domain experts affect the quality of decisions — has remained unexplored. We address this gap in this paper.

The second reason is that in many domains, the outcome of one uncertainty creates the source of another uncertainty. For the IBM case, for instance, the production team-lead recognized that she faced a two step uncertainty — whether new circuits will be complex or simple (with a 50% chance), and if their yield will be higher than old circuits. The outcome of the second uncertainty was contingent on the outcome of the first uncertainty (if the circuits are complex (simple) there is a 25% (75%) probability that the yield will be high). She could reduce this compound risk specification into the simple risk specification that there is a 50% chance that the yield of the new circuits will be high. This example is representative of the electronic device industry where each new product contains innovations on multiple dimensions/subparts. In project management, a manager may need to consider a two-step uncertainty that could lead to a delay — a highly efficient worker in charge of a task may leave the firm; and if that happens it may not be possible to find a second worker with the same efficiency resulting in a delay. The manager could provide a compound risk specification based on the outcomes {worker does not quit; worker quits and replacement is found; worker quits and replacement is not found} or she could provide simple risk specification based on the outcomes {no delay; delay} where the no delay option is an aggregation of the events {worker does not quit; worker quits and replacement is found}. In other domains, Elabed and Carter (2015) discusses the presence of compound risk in crop insurance markets where farmers buy insurance policies in which the payment is tied to two partially correlated indices.

## 1.2 Research Questions and Contributions to Literature

Simple risk specifications are appealing because they are less complex. Furthermore, in the subjective expected utility (SEU) framework, both specifications are equivalent and result in the same expected utilities from the project. As illustration, consider the IBM problem in which the probabil-

ity of success for the processor and heat sink under simple risk specification is equal to 0.5 and 0.3 respectively; assume the payoff is \$1000 if the project is successful on both dimensions, or else the payoff is \$0. It follows that the expected utility under the simple risk specification is equal to

$$E.U.(\text{Simple Risk}) = 0.5 \times 0.3 u[1000] + (1 - 0.5 \times 0.3)u[0],$$

where  $u[\cdot]$  is the utility function of the project manager. Now consider the compound risk specification provided by the team-leads that the probability of success of the processor is equal to (0.25,0.75) with 1/2 chance each, and assume that the probability of success of the heat sink is equal to (0.2,0.3,0.4) with 1/3 chance each. The expected utility under the compound specification is equal to

$$\begin{aligned} E.U.(\text{Compound Risk}) &= \frac{1}{2} \times \frac{1}{3} [(0.25 \times 0.2 + 0.25 \times 0.3 + 0.25 \times 0.4)] u[1000] \\ &\quad + \frac{1}{2} \times \frac{1}{3} [(0.75 \times 0.2 + 0.75 \times 0.3 + 0.75 \times 0.4)] u[1000] \\ &\quad + \left(1 - \left(\frac{1}{2} \times \frac{1}{3} [(0.25 \times 0.2 + 0.75 \times 0.3 + 0.75 \times 0.4)]\right)\right) u[0] \\ &\quad + \left(1 - \left(\frac{1}{2} \times \frac{1}{3} [(0.75 \times 0.2 + 0.75 \times 0.3 + 0.75 \times 0.4)]\right)\right) u[0], \end{aligned}$$

which reduces to

$$E.U.(\text{Compound Risk}) = 0.5 \times 0.3 u[1000] + (1 - 0.5 \times 0.3)u[0].$$

Thus, the expected utility under compound risk specification is identical to the expected utility under simple risk specification. Due to this equivalence, one might argue that the two risk specifications provided by the team leads are equivalent and are substitutes. But, behaviorally, substituting the simple risk specification for the compound risk specification is judicious only if doing so does not deteriorate the quality of a manager's valuation for the project. The prior literature does not provide evidence for whether this deterioration exists in multi-risk environments. Accordingly, we address the following closely related questions in this paper:

1. *Are managerial valuations for a project different under the two risk specifications?*
2. *If managerial valuations are different, then what behavioral factors drive these differences?*
3. *Which risk specification leads to more accurate valuations?*

We conduct laboratory experiments in which decision makers are provided with the probabilities of success of a project on two dimensions, in simple and compound risk specifications. We find that the valuations under the two risk specifications are statistically different. In our main result we show that the widely reported overestimation of conjunctive probabilities under simple risk specification reduces by a large extent under a compound risk specification. We also show that this mitigation occurs because individuals tend to exhibit aversion towards events whose probability of occurrence

is uncertain. This bias leads them to make valuation judgments *as if* the probability of success has lowered. Effectively, we show that the two biases in processing conjunctive events and compound risk specifications act in opposite directions, and as outcome, judgmental valuations for a two-dimensional project are closer to the true value with compound risk specification (versus a simple risk specification). Our findings imply that substituting compound risk specifications with simple risk specifications can lead to a systematic deterioration in the quality of project valuations.

Finally we note how our results contribute to the prior literature. There is a large body of literature on behavioral biases in decision making under uncertainty, including seminal works of Tversky and Kahneman (1974) and Kahneman and Tversky (1973). See Budescu and Fischer (2001) for a nice discussion on these biases and some common causes underlying these biases. Within this stream of literature, we focus on the intersection of the *number of risks* underlying risky cash flows and the *specifications* of the risks. Prior literature has looked how risk specifications for *one* risk affect the valuation of cash flows. This literature shows that individuals have a lower valuation for cash flows when the probability of success has a compound risk specification as compared to the case when the probability is specified using a simple risk specification (Keller, 1985; Segal, 1990; Harrison et al., 2015). Prior literature has also investigated managerial judgments for conjunctive events in which payments depend on obtaining specific outcomes simultaneously on *multiple* risks and each risk has a simple specification (Bar-Hillel, 1973; Yates and Carlson, 1986; Arkes, 1991). A consistent finding in these articles is that decision makers overestimate the probability of conjunctive events, which then leads to an overestimation of risky cash flows from the conjunctive events. But this existing body of literature has not studied managerial judgments for cash flows involving multiple risks with a compound risk specification, for conjunctive events. Further, there is no empirical evidence to suggest *a-priori* that decision makers will overestimate or underestimate the probability of success and the cash flows from conjunctive events under a compound risk specification. Our paper provides the first set of experimental results on this issue.

We proceed as follows. In Section 2, we develop research hypotheses. In Section 3, we discuss the experiment’s design and protocol. The data collected in the experiment and their analyses are in Sections 4 and 5. Section 6 contains a discussion on our findings and directions for future research.

## 2 Terminology and Hypotheses Development

### 2.1 Terminology

We first define the terminology used in the paper. Specifically, we define two types of events—*antecedent* and *conjunctive*—and two types of risk specifications—*simple* and *compound*. We consider a project with its cash flow contingent on the outcomes of two discrete random variables  $X$  and  $Y$ . Each random variable has two outcomes: *success* or *failure*. These outcomes are denoted as  $A$  and  $\backslash A$ , respectively, for  $X$ , and  $B$  and  $\backslash B$  for  $Y$ . The probability of success for the two random variables is denoted as  $Pr(A)$  and  $Pr(B)$ , respectively. We assume that the two random variables are mutually independent. It is standard in the probability literature to use the term *event* for

a set of outcomes. In our case we are interested in the outcome *success*. Accordingly, we define two events,  $A$  and  $B$ , and call these events the antecedent events. Therefore, the antecedent event  $A$  represents success on the first dimension of the project, and the antecedent event  $B$  represents success on the second dimension of the project. The simultaneous occurrence of  $A$  and  $B$ , denoted as  $A \cap B$ , is a conjunctive event, and it represents success on both dimensions. It follows from the assumption of mutual independence that the conjunctive probability of simultaneous success  $A \cap B$  is equal to  $Pr(A \cap B) = Pr(A) \times Pr(B)$ . The antecedent probabilities have discrete distributions over  $i = 1, 2, \dots, m_1; j = 1, 2, \dots, m_2$  points:

$$Pr(A) = w_i^A \text{ with probability } p_i^A \quad \forall i \text{ s.t. } \sum_i p_i^A = 1 \quad (1)$$

$$Pr(B) = w_j^B \text{ with probability } p_j^B \quad \forall j \text{ s.t. } \sum_j p_j^B = 1 \quad (2)$$

**Definition 1** (Simple Risk Specification). *Let  $p_i^A = 0 \quad \forall i \neq 1$  and  $p_j^B = 0 \quad \forall j \neq 1$ . Then, the antecedent events  $A$  and  $B$  have simple risk specifications.*

**Definition 2** (Compound Risk Specification). *Let  $p_i^A \neq 0$  for  $i = 1$  and at least one other  $i$ , and  $p_j^B \neq 0$  for  $j = 1$  and at least one other  $j$ . Then, the antecedent events  $A$  and  $B$  have compound risk specifications.*

In a simple probability specification, an event's probability of occurrence is known with certainty, and in a compound specification, the probability itself has a distribution. For example, consider the specification of probability of event  $A$ :  $[w_1^A, w_2^A] = [0.25, 0.75]$ . Now, when the weights are  $[p_1^A, p_2^A] = [1, 0]$ , the distribution of  $Pr(A)$  has a simple risk specification. When the weights are  $[p_1^A, p_2^A] = [1/2, 1/2]$ , the distribution of  $Pr(A)$  has a compound risk specification.

## 2.2 Hypotheses Development

We now develop the research hypotheses. The first two hypotheses pertain to decision makers' *absolute estimates* for (i) the conjunctive probability  $Pr(A \cap B)$  that the project will be successful on both its dimensions, and (ii) the valuation of risky cash flows from the project, under compound risk specifications for events  $A$  and  $B$ . The existing literature has not studied managerial judgments for conjunctive events with compound risk specifications; but prior studies on two extreme cases suggest contradictory directions. Accordingly, we set up two competing hypotheses.

The first case is judgments for cash flows from a conjunctive event  $Pr(A \cap B)$  when the antecedent events  $A$  and  $B$  have simple risk specifications. For this case, the prior literature including Cohen and Hansel (1957); Bar-Hillel (1973); Yates and Carlson (1986); Arkes (1991); Nilsson (2008) has consistently found that decision makers overvalue cash flows from conjunctive events and overestimate conjunctive probabilities. Bar-Hillel (1973) documents this overestimation in a laboratory experiment in which the decision makers could choose between two lotteries. In the first lottery, the payoff would be obtained only if a conjunctive event occurred: a coin would be flipped several times, and

the decision maker would obtain a payoff if all flips were successful—i.e., if the coin landed on Heads every time. In the second lottery, the same payoff would be dependent on a single outcome: a different coin would be flipped only once, and the decision maker would again receive a payoff if the flip were successful. The probabilities of obtaining Heads were different in both lotteries, but were such that the second lottery was preferable to the first. Yet a majority of the participants selected the first lottery. Since the payoff was the same in both lotteries, the preference for the first lottery was attributed to an overestimation of conjunctive probability in that lottery. Subsequent articles suggest that the overestimation may be due to the use of specific heuristics by decision makers. In one such widely recognized heuristic, subjects select one antecedent probability, say  $Pr(A)$ , and then insufficiently adjust this probability downward to account for the simultaneous occurrence of event  $B$  (Arkes, 1991). Another suggested heuristic is that decision makers tend to take an average of the two probabilities  $\frac{Pr(A)+Pr(B)}{2}$ , which always exceeds the true conjunctive probability  $Pr(A) \times Pr(B)$  (Yates and Carlson, 1986). Nilsson (2008) and Winman et al. (2014) also suggest that miscalculation in probability is responsible for the overestimation.

Our context is similar in that the success of the project entails a simultaneous success on both dimensions. Given the salience of the overestimation bias documented in the aforementioned prior literature for conjunctive events with simple risk specification, we would expect the overestimation in the probability of success to persist even with compound risk specification. Further since the valuations of risky cash flows depend on the perceived probabilities of receiving payoffs, the overestimation in decision makers' estimates of conjunctive probabilities are also likely to carry over to their valuation of risky cash flows from conjunctive events. This leads us to our first hypothesis:

**Hypothesis 1.** *In the presence of compound risk specification for each antecedent event that a project will be successful on each dimension, decision makers overestimate (i) the probability of the conjunctive event that the project will be successful simultaneously on both dimensions; and (ii) the valuation of risky cash flows from the project.*

The second case investigated in the prior literature is judgments for cash flows from antecedent events  $A$  and  $B$  with uncertain probabilities. The literature has explored two variants of this uncertainty. In the first variant, the probability of an antecedent event is unknown (Ellsberg, 1961). In the second variant the probability is uncertain but has a known distribution—i.e., the probability has a compound specification (Keller, 1985; Segal, 1990; Abdellaoui et al., 2015; Elabed et al., 2013; Harrison et al., 2015; Elabed and Carter, 2015). A common finding in the literature is that in the presence of compound risk for an antecedent event, decision makers' valuation of risky cash flows is less than the true value. This failure to reduce compound lotteries to simple lotteries has been attributed to several behavioral phenomena. Some articles (e.g. Keller, 1985; Segal, 1990; Budescu and Fischer, 2001) suggest that decision makers do not perform the correct mathematical calculations over compound risk specifications that are necessary to reduce them to equivalent risk specifications. Some other articles (e.g. Abdellaoui et al., 2015) suggest another behavioral reason – decision makers react against the possibility that the probability of their payoffs itself might be uncertain. This aversion has also been documented outside laboratory setting. For example, recently Elabed and Carter



(2015) discussed data for crop insurance plans purchased by farmers in Mali under a compound risk. The insurance plan is based on two correlated indices, (i) a farmer's own yield in his field, and (ii) the average yield from a number of fields in the region with the same climate/geography, including his own field. The compound risk from the two indices is as follows. There is some probability that the farmer's yield would be below a threshold. If that happens, his payment would be contingent on the regional yield, which is partially based on his yield, also being lower than a threshold (this is necessary to avoid moral hazard problems where a farmer chooses to not invest any effort in his field). The farmer needs to weigh this two step uncertainty to determine the probability that he will receive a payment under the insurance, and then determine his perceived fair price for the insurance. Data collected in Mali suggest that a majority of farmers did not buy the insurance at the fair price. Barham et al. (2014) discuss a similar decision making situation for buying production technologies.

In our context, decision makers would need to process compound risk information for two antecedents,  $A$  and  $B$ , and therefore the biases present in the valuation of these antecedent events due to compound risk specification may persist in judgments for their simultaneous occurrence (conjunctive event  $A \cap B$ ), as well. This leads us to our second hypothesis:

**Hypothesis 2.** *In the presence of compound risk specification for each antecedent event that a project will be successful on each dimension, decision makers underestimate (i) the probability of the conjunctive event that the project will be successful simultaneously on both dimensions; and (ii) the valuation of risky cash flows from the project.*

Our final hypothesis pertains to *directional* changes in the perceived probability of success and valuations for the conjunctive event  $A \cap B$  when risk specifications for the probabilities  $Pr(A)$  and  $Pr(B)$  change from simple to compound. No prior experiments in the behavioral decision-making literature address this specific issue for conjunctive events, but articles on antecedent events suggest a potential directional change. In one such recent article Abdellaoui et al. (2015) report results from an experiment in which decision makers provided responses for two sets of choices. In both sets, the participants chose between a lottery with a risky payoff and a certain amount. In the first set, the risky lottery had one urn with a specific number of red and black balls. To determine the risky payoff, one ball was drawn at random and a payment was made if the ball was red. In the second set, there were two urns with red and black balls. First, one urn was chosen at random, and then one ball was drawn from the selected urn. A payment was made if the ball drawn eventually was red. Essentially, the urn in the first set had a simple risk specification, while the urn in the second set had a compound risk specification. The two sets were such that the risky cash flows were identical in their outcomes (net payoff and probability of payoff). Yet participants adjusted their estimates downwards for the valuation of risky cash flows in the second set. Since the risky cash flows were identical in the two sets, the downward adjustment was attributed to a change from a simple risk specification to a compound risk specification.

In our context of conjunctive events, decision makers are exposed to two changes in risk specifications (one each for antecedent event  $A$  and  $B$ ) and, therefore, we expect the directional change in

judgments for antecedent probabilities and risky cash flows from antecedent events to persist for the conjunctive event  $A \cap B$ . Accordingly, we propose:

**Hypothesis 3.** *As compared to simple risk specifications for antecedent events (that a project will be successful on each dimension), decision makers' estimates are lower for (i) the probability of the conjunctive event that the project will be successful on both dimensions simultaneously; and (ii) the valuation of risky cash flows from the project, under compound risk specifications.*

We conducted a behavioral experiment, discussed next, to test these hypotheses.

### 3 Experiment Details

We first discuss the risk information shown to participants (Section 3.1), followed by the tasks that participants performed in the experiment (Section 3.2) and the experimental protocol (Section 3.3).

#### 3.1 Presenting Simple and Compound Risk Specifications

Participants were shown two types of urns on the computer: simple ( $R$ ) urns, with a simple risk specification (whose exact composition is known to the participants); and compound ( $C$ ) urns, with a compound risk specification (whose composition “process” is known). Figure 1 (a) shows the simple urns used. Urns  $R_1$  and  $R_2$  contains eight balls each. In urn  $R_1$ , four balls are black and four are white. In urn  $R_2$ , two balls are black and six are white. The black and white balls represent success and failure of a project on the first dimension, respectively. To determine the outcome from a specific urn, one ball is drawn randomly. Therefore, in urn  $R_1$ , there is a 50% chance that the project will be successful on that dimension. Urns  $R'_1$  and  $R'_2$  show the urns with the risk information for the second dimension. These two urns also have eight balls each. Urns  $R'_1$  and  $R'_2$  have four and two blue balls, respectively; and four and six yellow balls, respectively. In these urns, the blue and yellow balls represent successful and unsuccessful outcomes, respectively, on a second dimension (hence different colored balls). To determine whether the project is successful on this dimension, one ball is drawn randomly from the urn. These urns represent the situation where a manager provided the following estimates for the probability of success on the two dimensions of project: 50% in urns  $R_1$  and  $R'_1$ , and 25% in urns  $R_2$  and  $R'_2$ .

Figure 1 (b) shows the urns with compound risk specifications of success. To represent compound risk, the composition of the urn from which a ball will be drawn (for a successful outcome) is random. Specifically, we consider two possible urns in  $C_1$  for the first dimension of the project. The first of these two urns contains two black and six white balls, while the second urn contains six black and two white balls. One of these urns is chosen at random, and then a ball is drawn at random from the selected urn. If the ball drawn is black, the outcome is considered a success. Urn  $C'_1$  shows the compound risk specification on the second dimension, and, as earlier, a blue ball represents a successful outcome of the project on this dimension and a yellow ball represents a failure. Both urns  $C_1$  and  $C'_1$  show the situation in which a manager estimates that the project will be successful on

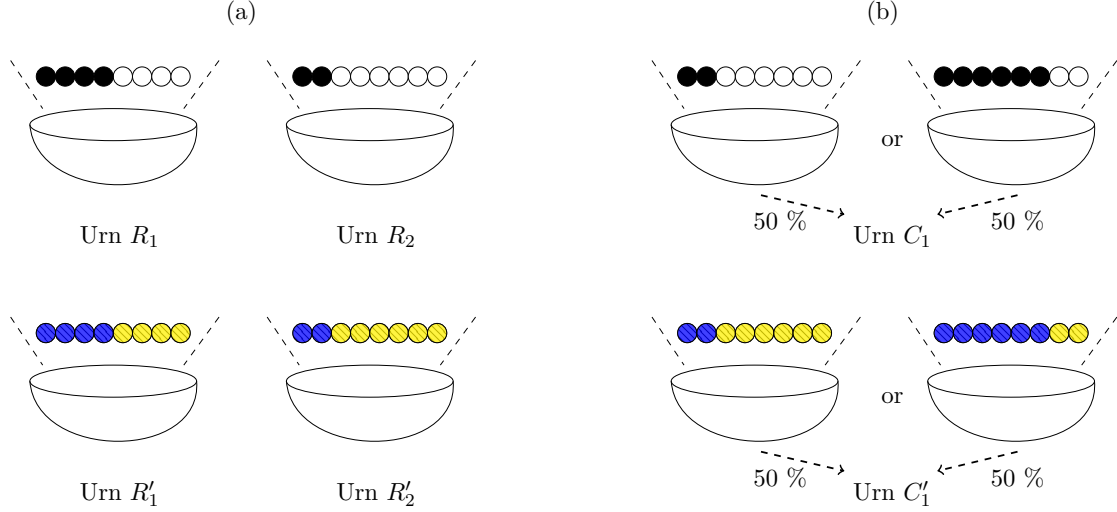


Figure 1: (a) Urns for simple risk specifications. Urn  $R_1$  contains four black and four white balls. Urn  $R_2$  contains two black and six white balls. One ball will be drawn from these urns. Urns  $R'_1$  and  $R'_2$  have blue balls instead of black and yellow balls instead of white. (b) Urns for compound risk specifications. For  $C_1$ , we have two urns: one with two black and six white balls; and one with six black and two white balls. We will first choose on urn at random and then draw a ball from the selected urn. The compound urn  $C'_1$  is similar, but with blue balls instead of black and yellow balls instead of white.

the two dimensions of the project with a 50% chance, but, on more careful reflection, specifies that these probabilities could be 1/4 or 3/4 with equal chances.

## 3.2 Decision Tasks and Motivation for Choosing them

### 3.2.1 Tasks: Antecedent and Conjunctive Events, under Simple and Compound Risk Specifications

There were ten rounds in total in the experiment. For our purpose, we focus on the data collected in six of these ten rounds. The data from the remaining four rounds are shown in Appendix Figure D.2. In each round we showed participants antecedent or conjunctive events, with simple or compound specifications, using various combinations of the urns shown in Figure 1.<sup>2</sup>

Three of these rounds were based on antecedent events ( $R_1, R_2, C_1$ ). In each of these three rounds, the participants were shown the corresponding urn(s) and the menu shown in Figure 2. Each of the ten pairs in the menu had two choices: a risky choice in which the payment was based on the color of the ball drawn randomly from the urn(s) shown in that round, and a safe choice with a certain cash flow. The certain cash flow varied from \$9 to \$18 in the ten pairs, in increments of \$1. The risky cash flow was the same in all ten pairs: a payment of \$24 if the ball drawn was black, and \$8 otherwise.

Three rounds were based on the conjunctive events:  $R_1 \& R'_1$ ,  $R_1 \& C'_1$ , and  $C_1 \& C'_1$ . In each of these

<sup>2</sup>(Figure D.1 in the Appendix contains visual representation for the four urns not included in the main text.)

The outcome of the lottery is based on the color of the ball that will be drawn from urn  $i$ .  
Please choose between Options G and H for each question.

	Option G	Option H
1)	\$24 if BLACK is drawn from Urn $i$ \$8 otherwise	\$ 9
2)	\$24 if BLACK is drawn from Urn $i$ \$8 otherwise	\$ 10
3)	\$24 if BLACK is drawn from Urn $i$ \$8 otherwise	\$ 11
4)	\$24 if BLACK is drawn from Urn $i$ \$8 otherwise	\$ 12
5)	\$24 if BLACK is drawn from Urn $i$ \$8 otherwise	\$ 13
6)	\$24 if BLACK is drawn from Urn $i$ \$8 otherwise	\$ 14
7)	\$24 if BLACK is drawn from Urn $i$ \$8 otherwise	\$ 15
8)	\$24 if BLACK is drawn from Urn $i$ \$8 otherwise	\$ 16
9)	\$24 if BLACK is drawn from Urn $i$ \$8 otherwise	\$ 17
10)	\$24 if BLACK is drawn from Urn $i$ \$8 otherwise	\$ 18

Figure 2: Decision Tasks for  $R_1$ ,  $R_2$  and  $C_1$ . In the experiment, we named the options G and H as options A and B. We have switched to using options G and H in the paper to avoid confusion with antecedent events defined earlier.

three rounds, the participants were shown the corresponding urns—for example, the urns  $R_1$  and  $R'_1$  for the conjunctive event  $R_1 \& R'_1$ , and the menu consisting of 10 pairs of choices as shown in Figure 3. The safe choices varied from \$9 to \$18, identical to the values in the rounds for antecedent events. The second choice involved a payment of \$24 if the ball drawn from the first urn in the pair was black **and** the ball drawn from the second urn in the pair was blue, \$8 otherwise. We note that our use of a multiple price list approach is consistent with the prior literature (e.g., Holt and Laury, 2002; Harrison and Rutstrom, 2008).

In each round, we expected (and found) participants to choose the risky cash flow in pair 1 and the certain cash flow in pair 10, and switch somewhere between. Our experimental setup asked the participants to specify the switch using a horizontal bar and placing it between two pairs. Suppose that a participant placed the bar between pairs 6 and 7; this meant that she chose the risky cash flow in pairs 1, 2, 3, 4, 5 and 6; and the certain cash flow in pairs 7, 8, 9, and 10. This setup avoids multiple switches by participants, as reported in some articles that have used multiple price lists (Harrison and Cox, 2008). Once a participant placed this bar, she was told the choices she had made on each of the ten pairs (see an example in Figure B.1 in the Appendix B). If she was satisfied with her choices in the ten pairs, she confirmed her choices and moved to the next round. Otherwise, she revisited her choices until she was satisfied with her selection.

The outcome of the lottery is based on the color of the ball that will be drawn from urn  $i$  **and** the color of the ball that will be drawn from urn  $j$ . Please choose between Options G and H for each question.

	Option G	Option H
1)	\$24 if BLACK is drawn from Urn $i$ <b>and</b> BLUE is drawn from Urn $j$ \$8 otherwise	\$ 9
2)	\$24 if BLACK is drawn from Urn $i$ <b>and</b> BLUE is drawn from Urn $j$ \$8 otherwise	\$ 10
3)	\$24 if BLACK is drawn from Urn $i$ <b>and</b> BLUE is drawn from Urn $j$ \$8 otherwise	\$ 11
4)	\$24 if BLACK is drawn from Urn $i$ <b>and</b> BLUE is drawn from Urn $j$ \$8 otherwise	\$ 12
5)	\$24 if BLACK is drawn from Urn $i$ <b>and</b> BLUE is drawn from Urn $j$ \$8 otherwise	\$ 13
6)	\$24 if BLACK is drawn from Urn $i$ <b>and</b> BLUE is drawn from Urn $j$ \$8 otherwise	\$ 14
7)	\$24 if BLACK is drawn from Urn $i$ <b>and</b> BLUE is drawn from Urn $j$ \$8 otherwise	\$ 15
8)	\$24 if BLACK is drawn from Urn $i$ <b>and</b> BLUE is drawn from Urn $j$ \$8 otherwise	\$ 16
9)	\$24 if BLACK is drawn from Urn $i$ <b>and</b> BLUE is drawn from Urn $j$ \$8 otherwise	\$ 17
10)	\$24 if BLACK is drawn from Urn $i$ <b>and</b> BLUE is drawn from Urn $j$ \$8 otherwise	\$ 18

Figure 3: Decision Task for  $R_1 \& R'_1$ ;  $R_1 \& C'_1$ ; and  $C_1 \& C'_1$ .

### 3.2.2 Remarks on Design

The design enables us to separate the impact of risk specifications on managerial judgments for antecedent events and for conjunctive events as follows:

1. Urns  $R_1$  and  $C_1$  are normatively equivalent, as both have a 50% probability of success with a payment of \$24, and a payment of \$8 otherwise. Therefore, differences between the switch points in these two rounds are due to the difference between a simple risk specification (in  $R_1$ ) versus a compound risk specification (in  $C_1$ ) for antecedent events.
2. Urns  $R_2$  and  $R_1 \& R'_1$  are normatively equivalent, as both have a 25% probability of success with a payment of \$24, and a payment of \$8 otherwise. Therefore, differences between the switch points in these two rounds are due to a bias in processing conjunctive events.
3. Urns  $R_1 \& R'_1$ ,  $R_1 \& C'_1$ , and  $C_1 \& C'_1$  are normatively equivalent, as all have a 25% probability of success with a payment of \$24, and a payment of \$8 otherwise. Therefore, differences amount the switch points in these three rounds due to a progressive change in risk specification from simple to compound for conjunctive events.

### 3.3 Experiment Protocol

The experiment was conducted in a laboratory setting at a large midwestern public university. The participant pool was comprised of undergraduate and graduate students in natural sciences, engineering, and business. The experiment was hosted on an online platform and consisted of three

parts. In the first part, we showed illustrative urns to explain how balls would be drawn from urns with simple and compound risk specifications and how the participants' payoffs were dependent on the outcomes of urns if they chose risky cash flow over certain cash flow. We explained that if they were shown only one urn in a specific round (in the rounds for urns  $R_1, R_2, C_1$ ), their payoff was dependent on the outcome of one risk. They would obtain a payoff of \$24 if the ball drawn randomly from the urn of that round was black, otherwise the payment would be \$8. If they were shown two urns in a round (in the rounds for urns  $R_1 \& R'_1$ ,  $R_1 \& C'_1$ , and  $C_1 \& C'_1$ ), their payoff was dependent on the outcomes of two risks: They would receive \$24 if a black ball was drawn from the first urn and a blue ball was drawn from the second urn, otherwise they would receive \$8. We also explained to them the certain payoffs for each of the ten pairs, and how to use the horizontal bar to make their selection for all ten pairs in a round. At this point, they did not know the exact composition of the urns in the ten rounds in the subsequent part of the experiment.

The second part consisted of the ten rounds in which the urns were presented to each participant in random order, and the participants made their choices in each round. In the third part, the participants provided demographic information, and we determined their compensation. To determine the compensation, we randomly selected one of the ten rounds, and then further randomly selected one pair of choices from that round. If the participant chose the certain cash flow for that pair, she was paid that certain amount. If the participant chose the risky cash flow of that pair, then the computer would draw balls from the urns for that round and determine the payment based on the colors of the balls drawn. The minimum and maximum payments were \$8 and \$24, respectively. A total of 73 participants completed the experiment, with an average payment of \$14.63. Appendix A shows the complete set of instructions used in the experiment.

## 4 Raw Data and Analysis

The raw data and preliminary observations are discussed in Section 4.1. Participants' choices in the experiment directly provided us with their valuations in the form of their switch points, and we can perform statistical tests of significance on these switch points in a nonparametric framework. These tests are in Section 4.2. We seek to explain the observed differences in switch points in normatively equivalent rounds as stemming from systematic changes in beliefs for the conjunctive probability due to changes in risk specifications. In Section 4.3, we develop a structural estimation model to estimate participants' probability beliefs within a subjective expected utility framework.

### 4.1 Data and Preliminary Analysis

A preliminary analysis of the data shows that participants understood the experiment correctly and that their behavior is consistent with prior literature. The raw data collected in the experiment are shown in Figures 4 (a) and (b). These figures show the fraction of participants (on the y-axis) that selected the risky cash flows at various values of certain cash flows (on the x-axis). For example, in Figure 4 (a), the line for  $R_1$  shows that nearly all participants selected the risky cash flow from this

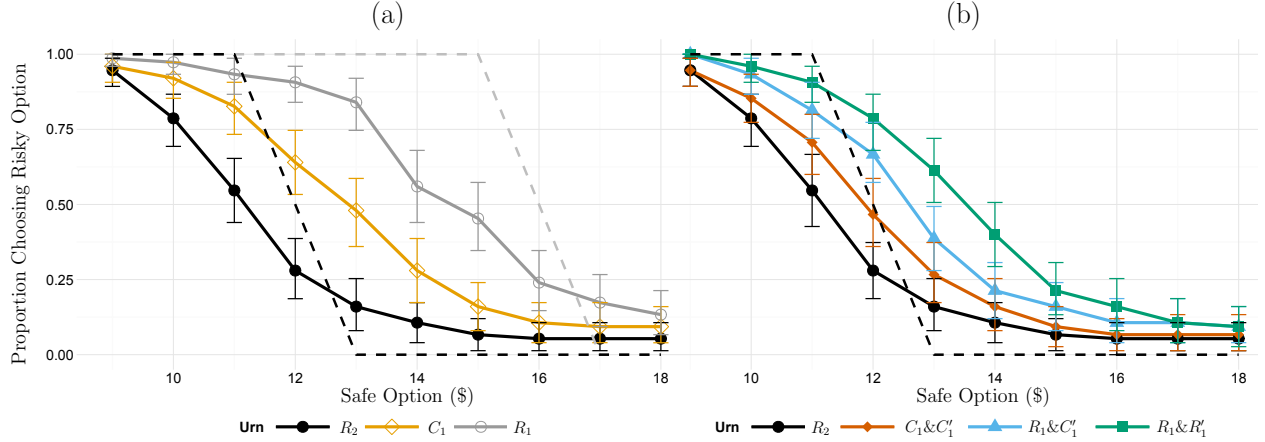


Figure 4: (a) Raw data for antecedent events. The left dashed line is the risk-neutral benchmark for the  $R_2$  Urn. The right dashed line is the risk-neutral benchmark for the  $R_1$  and  $C_1$  Urns. The vertical lines show the bootstrapped 95% confidence intervals. (b) Raw data for conjunctive events and for the urn  $R_2$  as reference. The dashed line is the risk-neutral benchmark for all four cases. The vertical lines show the bootstrapped 95% confidence intervals.

urn (in the round for the antecedent event  $R_1$ ) when the certain cash flow was \$10. As the certain cash flow payoff increased to \$13, \$16, and \$18, more participants opted for certain cash flows, and the fraction choosing the risky cash flow decreased to 85%, 25%, and 13%, respectively. Figure 4 (a) shows these fractions for the rounds for antecedent events  $R_1$ ,  $R_2$  and  $C_1$ , and Figure 4 (b) shows the same fractions for the rounds for conjunctive events  $R_1 \& R'_1$ ,  $R_1 \& C'_1$ , and  $C_1 \& C'_1$ .

Recall that, in each round, participants considered ten pairs; within each pair they selected between a risky cash flow and certain cash flow. The value of certain cash flow increased progressively in each pair, and they started to prefer the certain cash flow over the risky cash flow at a specific pair. They provided us with a *switch point* in their choices for that round using a horizontal bar. We define the switch point as the index of the last pair in which a participant chose the risky cash flow over the certain cash flow. For example, a switch point of 1 means that a participant placed the horizontal bar between pairs 1 and 2. He chose the risky cash flow over the certain cash flow of \$9 in pair number 1, but preferred the certain cash flows of \$10, \$11, ..., \$18 over risky cash flows in the next nine pairs. We assume that this selection also implies participant's *certainty equivalent* of \$9 for the urn in that round. Figure 5 shows aggregated statistics for the data within each round across all participants. The first column in the figure shows a benchmark: the switch point in a risk neutral framework. The second column shows the participants' average switch points. The last three columns show the 25th, 50th, and 75th quantiles of participants' switch points. Turning to the interpretation of numbers in the figure, as an example, the first row of numbers in Figure 5 shows that in the round in which they were shown urn  $R_1$  with an average risk neutral switch point of 7.5, participants switched to the certain cash flow at pair number 5.93, on average; and pair number 6 on median. These switch points correspond to certainty equivalents of approximately \$14, and are consistent with risk averse behavior.

Urn(s)	Risk Neutral Benchmark	Mean	St.Dev.	25th perc.	50th perc.	75th perc.
$R_1$	7.5	5.93 (0.26)	2.16 (0.19)	5 (0.23)	6 (0.58)	7 (0.32)
$R_2$	3.5	2.63 (0.18)	1.51 (0.14)	2 (0.47)	3 (0.5)	3 (0.48)
$C'_1$	7.5	4.19 (0.25)	2.16 (0.2)	3 (0.3)	4 (0.4)	5 (0.47)
$R_1 \& R'_1$	3.5	4.95 (0.24)	2.02 (0.18)	4 (0.48)	5 (0.25)	6 (0.23)
$R_1 \& C'_1$	3.5	4.1 (0.24)	2.05 (0.22)	3 (0.47)	4 (0.13)	5 (0.3)
$C_1 \& C'_1$	3.5	3.27 (0.22)	1.87 (0.19)	2 (0.36)	3 (0.29)	4 (0.43)

Figure 5: Switch Point Summary. The numbers represent the average switch point (among the ten pairs of options) in each round at which participants switched from risky to certain payoffs. Bootstrapped standard errors are shown in the parentheses.

Two features of the data suggest that participants understood the experiment correctly. Specifically, in the ten pairs in each round, the risky cash flow remains the same, while the certain cash flow increases from \$9 to \$18. We would expect each participant to switch from the risky cash flow to the certain cash flow as the value of certain cash flow increases. The switch point for each participant will depend on the participant's risk attitude. At an aggregated level, we would expect a growing fraction of participants to prefer the certain cash flow as the value of certain cash flow increases. We observe this trend in the data for all six rounds, as shown in Figure 4. Further, participants recognized the relative risks of simple events correctly. For example, urn  $R_2$  had a probability of success of 0.25, and urn  $R_1$  had a probability of success of 0.5. We see in Figure 4 (a) that at any level of certain cash flow, a larger fraction of participants chose the risky cash flow in the round for  $R_1$  than in the round for  $R_2$ , confirming that participants recognized the probability ordering of the two urns. For example, on average participants switched at pair number 5.93 for  $R_1$  and at pair number 2.63 for  $R_2$  (Figure 5).

Finally, the data show that the presence of compound risk specification introduced a downward bias in the certainty equivalents for antecedent events, consistent with Keller (1985). Specifically, both urns  $R_1$  and  $C_1$  had a 50% chance of success. Figure 4 shows that a consistently smaller number of participants chose the risky cash flow at all values of certain cash flow in the round for  $C_1$ , as compared to the round for  $R_1$ . Figure 5 shows that, on average, participants switched at pair number 5.93 for  $R_1$  and at pair number 4.19 for  $C_1$ .

## 4.2 Nonparametric Randomization Tests on Elicited Certainty Equivalents

We next conducted nonparametric randomization tests to investigate whether participants' average switch points for different rounds (that are normatively equivalent) exhibit a statistical difference. Nonparametric tests do not rely on any distributional assumption for the underlying random errors



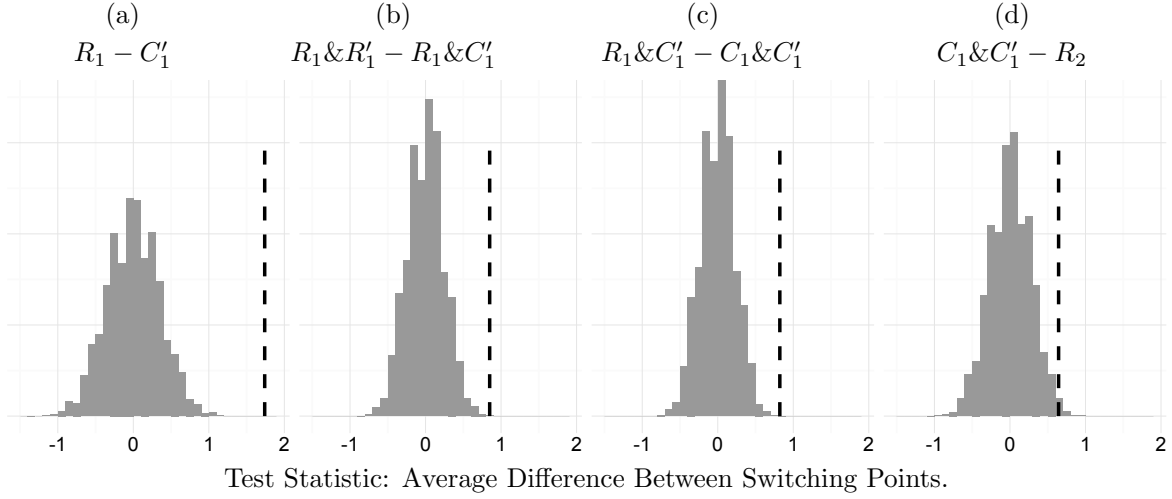


Figure 6: Randomization Tests. Distribution under the Null is in gray and actual observed outcome is marked by *dashed black* line.  $p$ -values: (a) 0.000, (b) 0.000, (c) 0.000, (d) 0.011.

and, therefore, provide stronger evidence for the documented differences. We report here the results of the tests conducted on four sets of rounds:  $R_1$  and  $C'_1$ ;  $R_1 \& R'_1$  and  $R_1 \& C'_1$ ;  $R_1 \& C'_1$  and  $C_1 \& C'_1$ ; and  $C_1 \& C'_1$  and  $R_2$ . Within each set, the first and the second rounds are normatively equivalent, in that the probability and the payoffs for the outcomes are identical for both rounds.

We now describe the randomization test on the example of participants' decisions for  $R_1 \& R'_1$  and  $R_1 \& C'_1$  rounds. As is the norm in randomization tests (see, e.g., Good, 2006), the null hypothesis is that participants' responses in these two normatively equivalent rounds are not different. If this is true, then the observed switch points in these two rounds are equally likely to have come from either  $R_1 \& R'_1$  or  $R_1 \& C'_1$ . That is, there two cases for each participant: (i) her responses in the two rounds are as she selected; and (ii) her responses in the two rounds are flipped. We sampled each participant and selected either (i) or (ii) for her with equal probability. Then we determined the average switch points for  $R_1 \& R'_1$  and  $R_1 \& C'_1$  across all participants, and computed the difference. We repeated this process 10000 times and obtained the histogram of the average differences (second panel of Figure 6). The vertical dashed line shows the actual average difference observed in the data for the two rounds. The  $p$ -value shows the probability of obtaining this extreme observation if the responses in the two rounds were actually interchangeable. The small  $p$ -value validates the presence of systematic differences between the certainty equivalents selected by participants in the two rounds,  $R_1 \& R'_1$  and  $R_1 \& C'_1$ . The remaining three graphs are interpreted similarly.

### 4.3 Aggregate Level Structural Estimation

The choices between certain cash flow and risky cash flow made by the participants stem from participants' risk attitudes and beliefs about the success probability for antecedents and conjunctive events, with simple and compound risk specifications. To deduce the risk attitudes and probability beliefs that best explain the data, we employed a latent structural model of choice. We estimated

the parameters of the model using likelihood maximization, and then used the likelihood ratio test to establish the statistical significance of various results.

#### 4.3.1 Estimation Details

We used the model considered in Harrison and Rutstrom (2008) to estimate parameters for choices under multiple price lists. We assumed that all participants  $j = 1, 2, \dots, 73$  represent various instances of a representative agent of the total population. The agent has a CRRA utility form:

$$u(x) = \frac{x^{1-\gamma} - 1}{1-\gamma}, \quad (3)$$

where  $x$  is the outcome and  $\gamma$  is the risk-aversion parameter to be estimated. Thus,  $\gamma = 0$  corresponds to a risk-neutral agent, and  $\gamma > (<)0$  corresponds to a risk-averse (risk-loving) agent.

The representative agent chooses the option with the highest expected value given her belief about the probability of success in round  $t$ ,  $p_t$ . We assumed that the beliefs about simple urns are correct (e.g., the belief about the  $R_1$  urn is  $p_{R_1} = 0.50$ ), which allowed us to estimate the risk aversion. But we did not assume the beliefs about the compound urns and conjunctive events to be correct and, instead, estimated these beliefs. In each round  $t$ , the participants considered ten pairs. For each pair, they chose between a risky cash flow, which we denote as option G (Figures 2 and 3), and a certain cash flow, which we denote as option H. The choice between options G and H is subject to an error, which is assumed to be distributed according to a logistic distribution centered at zero. Then, the probability with which Option G was selected for each pair of options is formulated as:

$$Pr_{G_{i,t}} = \frac{1}{1 + e^{-\lambda[E_{p_t}u(G_{i,t}) - u(H_{i,t})]}}, \quad (4)$$

where  $Pr_{G_{i,t}}$  is the probability that the representative agent chooses option G at round  $t$ ;  $t = 1, 2, \dots, 6$  for the choice pair  $i$ ;  $i = 1, 2, \dots, 10$ ;  $G_{i,t}$  and  $H_{i,t}$  are the  $i$ th choice pair presented to the participants in round  $t$ ;  $p_t$  represents the agent's belief for the probability of success in round  $t$ ; and  $\lambda$  is a parameter capturing the precision with which the agent makes a choice.<sup>3</sup> Finally, since the action space (choice between options G and H) is binary, we employed the following likelihood function to estimate the parameters  $\gamma, \mathbf{p}, \lambda$ :

$$\mathbf{L}(\gamma, \mathbf{p}, \lambda) = \prod_{i,j,t} Pr_{G_{i,t}}^{y_{i,j,t}} \times (1 - Pr_{G_{i,t}})^{(1-y_{i,j,t})}, \quad (5)$$

where  $\mathbf{p}$  is the vector of beliefs  $p_t$ ;  $Pr_{G_{i,t}}$  is defined in (4); the utility function  $u(\cdot)$  in (4) is defined in (3); and  $y_{i,j,t} = 1$  if participant  $j$  chose option G in pair  $i$  in round  $t$ , or else  $y_{i,j,t} = 0$ . We found the optimal values of the parameters by maximizing equation (5).

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<sup>3</sup>We normalize  $\lambda$  by the utility difference between the best and the worst case scenario as recommended in Wilcox (2011).

Parameter	<i>Unrestricted</i>	$p_{C_1} = .5$	$p_{R_1 \& R'_1} = p_{R_1 \& C'_1}$	$p_{R_1 \& C'_1} = p_{C_1 \& C'_1}$
$\gamma$	0.56 (0.02)	0.77 (0.02)	0.56 (0.02)	0.56 (0.02)
$p_{C_1}$	0.39 (0.01)	0.5	0.39 (0.01)	0.39 (0.01)
$p_{R_1 \& R'_1}$	0.42 (0.01)	0.46 (0.01)	0.4 (0.01)	0.42 (0.01)
$p_{R_1 \& C'_1}$	0.38 (0.01)	0.43 (0.01)	-	0.36 (0.01)
$p_{C_1 \& C'_1}$	0.35 (0.01)	0.39 (0.01)	0.35 (0.01)	-
$\lambda$	19.03 (0.56)	16.52 (0.49)	18.89 (0.56)	18.9 (0.56)
<i>LogLike</i>	1614.21	1734.75	1624.64	1624.41
<i>p-val.</i>	-	0	0	0

Figure 7: Aggregate Estimates. Estimates for risk aversion,  $\gamma$ , subjective beliefs,  $p$ , about the probability of success, and the precision parameter,  $\lambda$ . Standard errors are presented in the parentheses. The  $p$ -values are calculated using likelihood ratio test relative to the unrestricted model.

#### 4.3.2 Parameter Estimates and Interpretation

The parameter estimates obtained using the data are shown in Figure 7. Each pair of numbers in the Figure shows the estimated parameter value and the corresponding standard error in parentheses. The second column shows the unrestricted parameter estimates. This column shows that the aggregated agent's risk-aversion parameter is equal to  $\gamma = 0.56$ , which is consistent with prior experiments including Harrison and Cox (2008) and Moreno and Rosokha (2013). The representative agent believed the probability of success in the conjunctive event  $R_1 \& R'_1$  with simple risk specification to be  $p_{R_1 \& R'_1} = 0.42$ , and so on. The third column presents these estimates/beliefs when we force the assumption that the agent correctly estimated the compound probability of success in  $C_1$  at 0.5. The next two columns show parameter estimates under the assumptions that participants estimated the conjunctive probabilities to be equal in rounds  $R_1 \& R'_1$  and  $R_1 \& C'_1$ ; and in rounds  $R_1 \& C'_1$  and  $C_1 \& C'_1$ , respectively. The last row provides  $p$ -values for the statistical tests for whether these assumptions are supported by the data.

We are now ready to comprehensively test our hypotheses 1–3 for whether risk specifications interact with valuations of multi-dimensional projects.

## 5 Effect of Compound Risk Specifications on Judgmental Valuations of Multi-Dimensional Projects

### 5.1 Do Decision Makers Overvalue or Undervalue Projects Under Compound Risk Specifications?

We set up competing Hypotheses 1 and 2 to determine whether decision makers overvalue or undervalue a two-dimensional project and the probability of its success under compound risk specifications. Next we evaluate the evidence for testing these competing hypotheses.

#### 5.1.1 Valuation of Cash Flows

The data support an overvaluation of a two-dimensional project under compound risk specification. Specifically, both the conjunctive event  $C_1 \& C'_1$  (that the a two-dimensional project will be successful on both dimensions) and the antecedent event  $R_2$  (that the a one-dimensional project will be successful on its one dimension) have a probability of success of 0.25. In both events, the payoff is \$24 for success and \$8 otherwise. The antecedent event  $R_2$  is a benchmark: it has a simple probability of success of 0.25, and participants' valuations for  $R_2$  are subject to their individual risk attitudes. In contrast, their valuations for  $C_1 \& C'_1$  are subject to both their risk attitude and their two behavioral biases for conjunctive event and compound risk specification. Therefore, the difference in participants' valuations for  $C_1 \& C'_1$  relative to  $R_2$  is due to the additional presence of compound risk specification for conjunctive event in  $C_1 \& C'_1$ . Figure 4 shows that participants consistently selected the risky cash flow more frequently for  $C_1 \& C'_1$  than for  $R_2$  at various levels of certain cash flows. For example, in the round for  $R_2$ , approximately 78%, 55%, 27%, and 13% selected the risky cash flow at the certain payments of \$10, \$11, \$12, and \$14, respectively, but these fractions increased to 85%, 68%, 45%, and 17% in the round for  $C_1 \& C'_1$ .

Figure 5 further sharpens this finding. Participants, on average, switched from the risky cash flow to the certain cash flow at pair number 3.27 for the conjunctive event  $C_1 \& C'_1$ , but at the statistically different pair number 2.63 for the antecedent event  $R_2$ . This suggests that participants believed that the cash flow potential from  $C_1 \& C'_1$  to be higher than that from  $R_2$ . In the experiment, the pair number 3.27 corresponds to a certainty equivalent between \$11 and \$12 while the pair number 2.63 corresponds to a certainty equivalent between \$10 and \$11. The non-parametric randomization test in Figure 6 provides the strongest evidence. The rightmost panel shows that there is only a 1% chance that participants' in the two rounds intended to switch from the risky cash flows to certain flows at the same pair, and would display the differences observed in the data due to presence of random noise only.

#### 5.1.2 Overestimation of Probability of Success

The parametric estimates for probability beliefs provide evidence for a mechanism at play. The cash flows were identical in the rounds for  $C_1 \& C'_1$  and  $R_2$ , suggesting that the differences in the switch

points in these two rounds are due to the differences in the probability beliefs for these rounds. Figure 7 shows that after controlling for risk aversion, participants, on average, estimated the probability of success in the conjunctive event  $C_1 \& C'_1$  at  $p_{C_1 \& C'_1} = 35\%$ . With a standard error of 1%, this value is statistically different from the true value of 25% for the equivalent urn  $R_2$ .

Based on these results we fail to reject Hypothesis 1, but we do reject Hypothesis 2. We conclude that decision makers overestimate the value of risky cash flows and the probability of success of a project with two dimensions, when the risks have compound specifications.

Finally, we note a subtle but an important point. In the round  $R_2$  with simple risk specification, the average switch point in a risk neutral environment is 3.5. But participants on average switched at pair number 2.63. The gap between the switch points 3.5 and 2.63 is due to participants' risk aversion. For the equivalent conjoint event  $C_1 \& C'_1$  with compound risk specification, participants' average switch point is 3.27 which is higher than the average switch point of 2.63 in  $R_2$  but not statistically different from risk neutral switch point of 3.5. Therefore, when controlling for their risk aversion, participants did overestimate the cash flows from conjunctive event  $C_1 \& C'_1$ . However, their average valuation for  $C_1 \& C'_1$  is close to the normative value in a risk-neutral framework.

## 5.2 Does a change in Risk Specification from Simple to Compound Decrease Overestimation of Project Valuation?

We now consider differences in project valuations under the two risk specifications in order to test Hypothesis 3.

### 5.2.1 Decrease in Overvaluation When Risk Specification Changes

**Evidence from Average Behavior:** The evidence comes from the data for the three rounds based on conjunctive events:  $R_1 \& R'_1$ ,  $R_1 \& C'_1$ , and  $C_1 \& C'_1$ . These three conjunctive events are normatively equivalent: They have an equal probability of success at 25% and an equal payment of \$24 for the success outcome and \$8 otherwise. In these three rounds the risk specification changes from being a completely simple risk specification (in  $R_1 \& R'_1$  where the risks on both dimensions have simple risk specifications) to a partially compound risk specification in  $R_1 \& C'_1$  (the risk on the second dimension has a compound risk specifications) to a completely compound risk specification in  $C_1 \& C'_1$  (both dimensions have compound risk specifications). Therefore, the progressive differences in participants' valuations and probability judgments in the three rounds are due to a progressive change in risk specification from simple to compound. We remind the reader that these three rounds were shown to each participant in a random order. Therefore the patterns described below in participants' valuations for the three rounds are not subject to biases due to the order of presentation of the rounds.

The data for these three rounds in Figures 4, 5, and 6 show that participants' certainty equivalents lowered as the presence of compound risk specification increased. Figure 4 (b) shows that at any specific value of certain cash flow, the fraction of participants who preferred the risky cash flow was larger in  $R_1 \& R'_1$  than in  $R_1 \& C'_1$ ; and in  $R_1 \& C'_1$  than in  $C_1 \& C'_1$ . This trend is especially salient

around the risk-neutral switch point for these rounds. When \$11 was available for certain, nearly 90% participants preferred the risky cash flow in  $R_1 \& R'_1$ . In contrast, only 80% preferred the risky cash flow in  $R_1 \& C'_1$  and even fewer, nearly 70%, chose the risky cash flow in  $C_1 \& C'_1$ . When \$12 was available for certain, nearly 80% participants preferred the risky cash flow in  $R_1 \& R'_1$ , but only 70% and 45% chose the cash flow in  $R_1 \& C'_1$  and  $C_1 \& C'_1$ , respectively.

Figure 5 provides average switch points from the three rounds. As the figure shows, participants' average switch points in  $R_1 \& R'_1$ ,  $R_1 \& C'_1$ , and  $C_1 \& C'_1$  (from the risky cash flows to certain flows) changed from pair number 4.95 to pair number 4.10 to pair number 3.27, respectively. In the experimental setup, both switch points of pair numbers 4.95 and 4.10 lie between \$12 and \$13, while the switch point of pair number 3.27 corresponds to being between \$11 and \$12. Figure 6 provides nonparametric statistical confirmation that these differences in the switch points are statistically significant. Specifically, we see in the second and third panels that participants' average switch point in round  $R_1 \& R'_1$  exceed their average switch point in round  $R_1 \& C'_1$  (with a p-value of 0.00); and the average switch point in round  $R_1 \& C'_1$  exceed the average switch point in round  $C_1 \& C'_1$  (with a p-value of 0.00). This trend confirms that the average certainty equivalent were lower under compound risk specifications as compared to simple risk specification.

**Evidence from Median Behavior:** To ensure that our average behavior based analysis was not biased due to outliers in the data, we repeated the analysis using median behavior. The sixth column in Figure 5 shows the median switch points in the rounds  $R_1 \& R'_1$ ,  $R_1 \& C'_1$ , and  $C_1 \& C'_1$ . Similar to the trend observed in the average behavior, the median switch points for the three rounds decreased from pair number 5 to pair number 4 to pair number 3. Remarkably, the median switch point for  $R_2$  is also 3. This observation is important because the rounds  $R_1 \& R'_1$ ,  $R_1 \& C'_1$ , and  $C_1 \& C'_1$  are normatively equivalent to  $R_2$  but are subject to conjunctive probability bias and compound specification bias. Equal median switch points in  $R_2$  and  $C_1 \& C'_1$  imply that compound risk specification in both antecedent events brought the median of participants' evaluations for these events in sync, consistent with subjective expected utility framework. We also repeated the permutation tests for the median and observed similar results. The output of the median permutation tests is provided in the Appendix Figure C.1.

### 5.2.2 Decrease in Overestimation of Conjunctive Probability When Risk Specification Changes

The results in the second column of Figure 7 show that the trends in the certainty equivalents selected by participants in the three rounds,  $R_1 \& R'_1$ ,  $R_1 \& C'_1$ , and  $C_1 \& C'_1$ , can be explained by their beliefs for the probability of success of the project in the three rounds. The participants' subjective beliefs for the probability of success decreased as compound specifications replaced simple risk specifications, going from  $R_1 \& R'_1$  to  $R_1 \& C'_1$ , to  $C_1 \& C'_1$ . On average, their belief that the project would be successful on both dimensions was  $p_{R_1 \& R'_1} = 42\%$  in the round for  $R_1 \& R'_1$ . This number decreased to  $p_{R_1 \& C'_1} = 38\%$ , a statistically significant decline of 4%, in the round for  $R_1 \& C'_1$ , and it

decreased further to  $p_{C_1 \& C'_1} = 35\%$  for  $C_1 \& C'_1$ .

During the parameter estimation process, we also estimated the perceived probabilities assuming that participants believed that the probability of success in rounds  $R_1 \& R'_1$  and  $R_1 \& C'_1$  are equal, and that those in rounds  $R_1 \& C'_1$  and  $C_1 \& C'_1$  are equal. The estimated parameters for these two assumptions are in the third and fourth columns in the figure respectively. The last row in the two columns shows the results for the statistical tests for the assumptions. Both p-values are 0.000, leading us to conclude that (i) the data do not support these two assumptions, and (ii) the differences in participants' choices for certainty equivalents about the three rounds are consistent with the presence of different beliefs for the probability of success, with changes in risk specifications.

Based on this evidence, we fail to reject Hypothesis 3 and conclude that decision makers' estimates for (i) the probability that a project will be successful on both dimensions simultaneously; and (ii) the valuation of risky cash flows from the project, are lower under compound risk specifications as compared to simple risk specifications.

### 5.2.3 Can Random Errors Explain the Observed Behavior?

We conducted further analyses to rule out the possibility that the participants did not understand the compound setup correctly and made random errors. Specifically, if the introduction of compound risk increased the random noise in participants' responses, we would expect the average switch point to move towards 5 (the average among available switch points). But instead we observed a consistent trend away from 5: as the presence of compound risk specification increased from  $R_1 \& R'_1$  to  $R_1 \& C'_1$  to  $C_1 \& C'_1$ , the average switch point consistently decreased from 4.95 to 4.10 to 3.27, which is counter to what we would have expected under random errors. Thus, the conjecture that an increase in random behavior, which may be due to the increased complexity of compound urns, would lead to the observed behavior is negated by the data.

## 6 Discussion and Future Research

This research is motivated by the widespread presence of compound risk specifications in multi-risk environments, and a lack of understanding for how compound risk specifications affect project valuations when a successful outcome is required for all risks simultaneously. We set out to explore three interrelated questions: (i) do different risk specifications (simple/compound) lead to different valuations? (ii) what behavioral factors drive these differences? and (iii) which risk specification leads to higher quality valuations? Our experimental results provide clear directional answers for these questions, as well as insights and prescriptions for managerial decision making in multi-risks environments.

In answer to the first question, our results show that risk specifications systematically impact managerial valuation of multi-dimensional projects. Specifically, using compound risk specifications leads to a *smaller* gap between a project's true value and decision makers' perceived value, as compared to simple risk specifications. Our results show that this smaller gap is the *net* outcome

of two simultaneous behavioral biases during decision making for multidimensional projects with compound risk specifications. The first bias is the conjunctive probability bias that leads decision makers to overestimate the probability of a simultaneous success on both dimensions. The second bias is the compound risk specification bias that leads decision makers to behave as if the probability of success on an individual dimension has decreased.

Our experiment shows that both biases coexist. Furthermore, they both involve decision makers not correctly performing multiplication operations during estimation: in order to correctly determine the conjunctive probability, one needs to multiply the probability of success on each of the two dimensions and decision makers appear to not perform this calculation correctly; in order to correctly process the compound risk specification, one needs to calculate a weighted average of the distribution of probability values and decision makers appear to not perform this multiplication correctly either. Yet, as shown by our experiments, the two biases work in opposite directions. The conjunctive probability bias increases the perceived value and perceived probability of success of a project, while the compound probability bias decreases these perceived quantities.

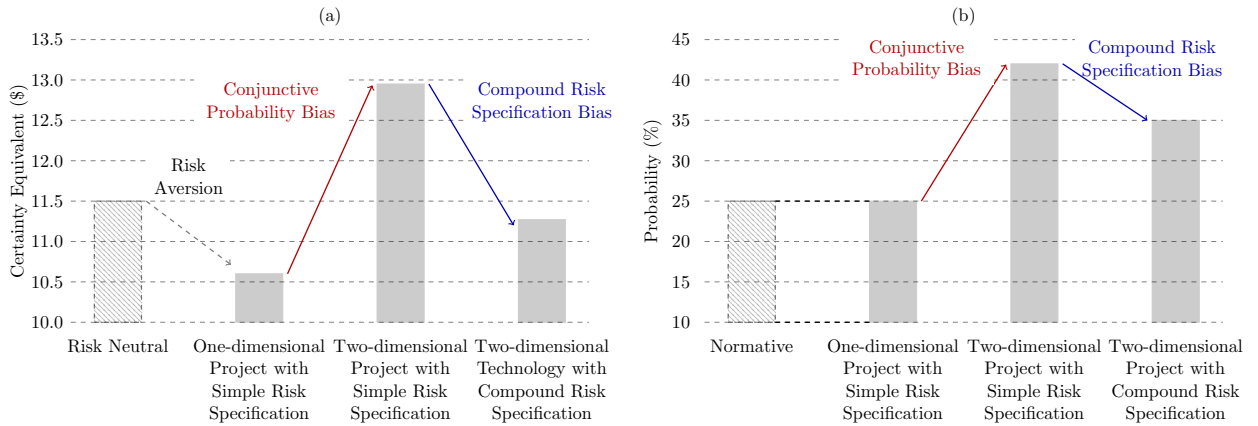


Figure 8: Conjunctive probability bias and compound risk specification bias act in opposite directions. (a) Trends in certainty equivalents. We have translated the switch points observed into dollar amounts for certainty equivalents. We also note that in our setup, a risk neutral rational participant is equally likely to switch to certain cash flow at pair number 3 or 4. The dollar amount \$11.5 shown in the figure is at the average switch point of 3.5. (b) Trends in probability beliefs. We assumed that agents form normative beliefs for one-dimensional project with simple risk specification, but estimated beliefs for two-dimensional project and for compound risk specification.

The compensatory effect of the two biases is salient in Figures 8 (a) and (b). Figure 8 (a) shows a normative benchmark and participants' responses in three normatively equivalent cases (rounds  $R_2, R_1 \& R'_1$ , and  $C_1 \& C'_1$ ) considered in our experiment. The first vertical bar shows the certainty equivalent for a risk neutral decision maker for these four cases. The second vertical bar shows the average certainty equivalent of the participants for one-dimensional project with simple risk specification. Comparing the heights of the two bars, it is evident that participants' average certainty equivalent is lower than the normative value, due to risk aversion. When the one-dimensional project changed to a two-dimensional project under simple risk specification, participants valued the two-



dimensional project higher as shown by the third vertical bar. This increase is due to conjunctive probability bias. When we provided participants with compound risk specification instead simple risk specification, participants' valuations were lower as shown by the fourth vertical bar. Remarkably, this last certainty equivalent is not statistically different from the risk-neutral valuation, suggesting that from a risk-neutral firm's perspective, compound risk specification could completely mitigate the impact of conjunctive probability bias as well as individual risk aversion in managerial judgments.

Figure 8 (b) shows that these changes in perceived valuations can be explained by changes in the perceived probability of success of the project. The first bar shows the true probability of success in all three cases, at 25%. The second bar shows the probability value to be the same under simple risk specification for a one-dimensional project. The perceived probability increases to 42% when the project needs to be successful on both dimensions with simple risk specification, as shown by the third vertical bar. But when the risk specification of two-dimensional project changed to compound risk specification, the perceived probability decreased again to 35%. These insights provide a nuanced understanding of the factors affecting valuations of compound risk specifications for conjunctive events, and answer the second question.

Finally, to answer the third question, our results imply that when compound risks specifications are available for multiple risks, they should not be substituted with equivalent simple risk specifications before communication to decision makers.

Our work leads to some natural extensions that should be explored in future research. We restricted our attention to two-dimensional projects. Extending our experimental approach and analysis to more than two dimensions will provide insights for managing multiple risks. While we expect the directional benefit of using compound risk specifications over simple risk specifications to persist, the magnitude of the benefit from using compound risk specifications may not remain the same. Behavioral experiments will provide a natural approach to quantify this magnitude. In addition, in our experiments, we considered compound distributions where each probability value itself had a two-point distribution (e.g., the probability of success could be 0.25 or 0.75 with equal chances). This setup was easily understood by participants in the experiment and also simplified the computation of reduced probabilities if the participants wanted to reduce the compound probabilities. Future research should investigate scenarios in which probabilities have compound distributions with more than two points. While we expect our main result—that compound risk specifications lead to more accurate valuations in multi-risk environments as compared to simple risk specifications — to hold true, establishing the relationship between the rate of decrease and the number of points in the compound risk specification will be useful.

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## Appendix A Experimental Instructions

Today's experiment will last about 30 minutes. Everyone will earn at least \$8.00 and a maximum of \$24.00. In total, you will face 10 tasks that affect your potential earnings. Each task will contain 10 decisions. At the end of the experiment, one of the decisions will be chosen randomly and carried out to determine your actual money earnings.

### Sample Task Description

In each task we will show you two urns and ask you to make 10 associated decisions. Each decision is a choice between two options, labeled Option A and Option B. You can only gain money in these tasks; you cannot lose any money.

**Option A:** The outcome of Option A is based on the color of the ball that will be drawn from Urn L AND the color of the ball that will be drawn from Urn R. The diagram below summarizes the urn composition.

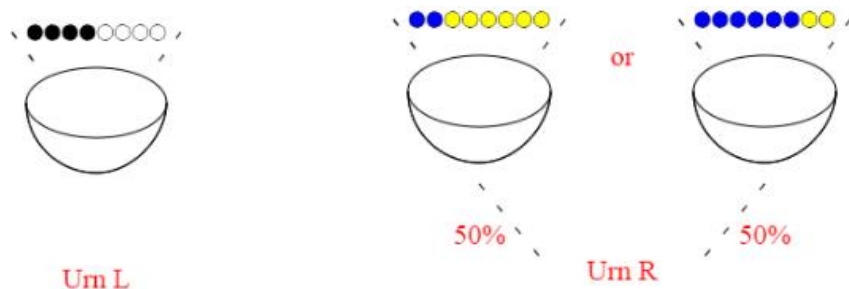


Figure A.1: Example Urn Description.

The composition of Urn L is known and the composition of the Urn R is uncertain. Specifically, Urn L has 4 BLACK and 4 WHITE balls and one ball will be drawn at random from this urn. Urn R has either 2 BLUE and 6 YELLOW balls, which happens with 50% probability, or 6 BLUE and 2 YELLOW balls, which happens with 50% probability. In other words, there is a 50-50 chance that one of the two urns presented below will be selected to be the Urn R and then one ball will be drawn at random from the selected urn.

**Option B:** The outcome of Option B is a fixed dollar amount.

For the urns showed on the earlier page, you will make 10 choices between Option A and Option B using a table presented below.

	Option A	Option B
1)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R. \$8 otherwise	\$9
2)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R. \$8 otherwise	\$10
3)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R. \$8 otherwise	\$11
4)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R. \$8 otherwise	\$12
5)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R. \$8 otherwise	\$13
6)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R. \$8 otherwise	\$14
7)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R. \$8 otherwise	\$15
8)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R. \$8 otherwise	\$16
9)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R. \$8 otherwise	\$17
10)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R. \$8 otherwise	\$18

Figure A.2: Example Decision Task.

Option A will be the same for each of the 10 decisions, while Option B will range from \$9 to \$18. Rather than selecting between options A and B separately for each decision, you will make a choice for all decisions simultaneously by clicking on the cut-off at which you choose to switch from Option A to Option B. For example, in the case that you prefer \$9 over Option A you can click just above the first decision. In the case that you prefer Option A over \$18 you can click just below the last decision.

## Number of Tasks

In total you will face 10 separate tasks. In each task you will be presented either with Urn L, Urn R, or both Urn L and Urn R (as in the previous example). In the case when you are presented with one urn, Urn L or Urn R, Option A will depend on the random draw from that urn. In the case when you are presented with two urns, Option A will depend on the random draw from both urns. Composition of urns may change from one task to another and therefore it is important to pay attention to the description of urns for each task.

Note that, urns are associated only with one task. In other words, urns in Task 1 will be independent from urns in Tasks 2, 3, ..., 10. At the end of the experiment the computer will carry out the random draws from urns for each task.

## Payment

In total you will make 100 decisions labeled 1, ...,100. At the end of the experiment computer will randomly draw a number from 1 to 100. This number will be the decision for which you will be compensated on. Then depending on the random draw from the corresponding urn and your choice you will get paid in cash.

## Appendix B Screenshot of the Interface

	Option A	Option B
81)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R \$8 otherwise	\$9
82)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R \$8 otherwise	\$10
83)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R \$8 otherwise	\$11
84)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R \$8 otherwise	\$12
85)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R \$8 otherwise	\$13
86)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R \$8 otherwise	\$14
87)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R \$8 otherwise	\$15
88)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R \$8 otherwise	\$16
89)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R \$8 otherwise	\$17
90)	\$24 if BLACK is drawn from Urn L and BLUE is drawn from Urn R \$8 otherwise	\$18

Summary: You choose Option A over Option B for decisions: 81,82,83,84  
You choose Option B over Option A for decisions: 85,86,87,88,89,90

If this is correct please click 'Submit'

Submit

Figure B.1: Decision Screen

## Appendix C Randomization Test for the Median

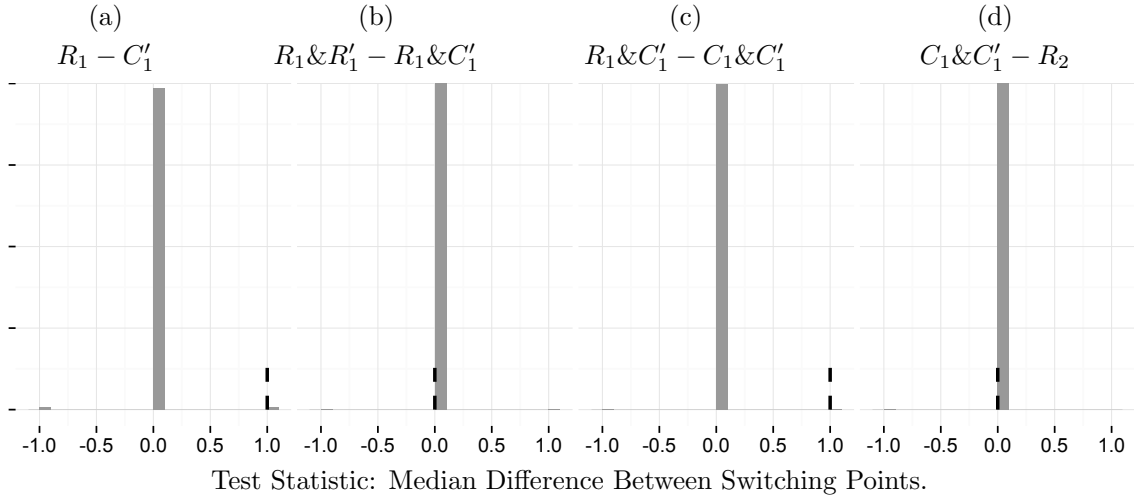


Figure C.1: Randomization Tests. *Notes:* Distribution of the Median under the Null is in *gray*. Actual observed outcome is marked by *dashed black* line. *p*-values: (a) 0.007, (b) 0.999, (c) 0.000, (d) 0.998.

## Appendix D Additional Decision Tasks

### D.1 Urn Construction

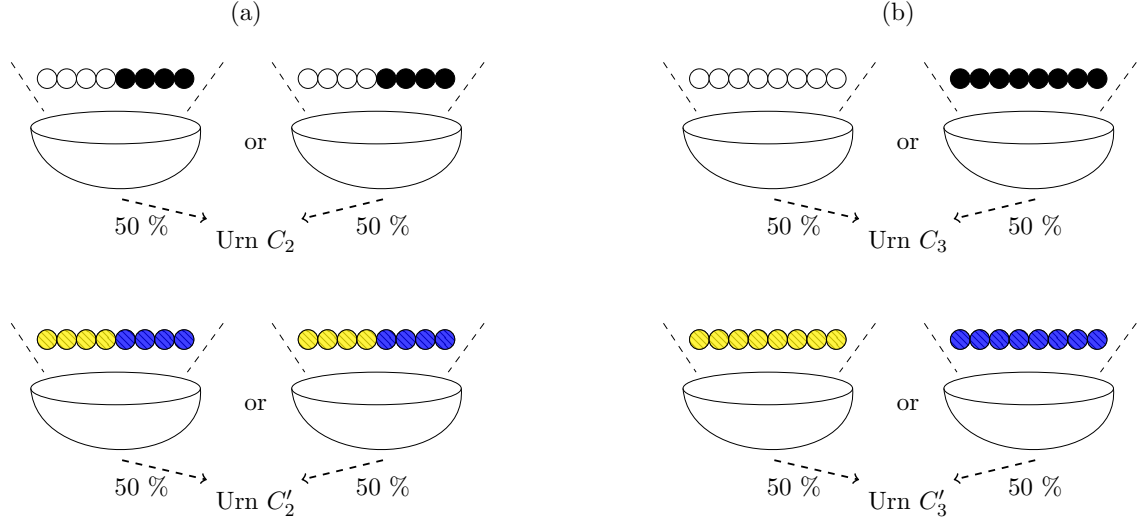


Figure D.1: **(a)** For  $C'_2$ , we will first choose randomly from two urns: one with four black and four white balls; and one with four black and four white balls. Then we will draw a ball from the urn selected. The compound urn  $C'_2$  is similar with blue balls instead of black, and yellow balls instead of white. **(b)** Urns for compound risk specification. For  $C'_3$ , we will first choose randomly from two urns: eight white balls; and one with eight balls. Then we will draw a ball from the urn selected. The compound urn  $C'_3$  is similar with blue balls instead of black, and yellow balls instead of white.

### D.2 Switch Point Summary

Urn(s)	Risk Neutral Benchmark	Mean	St.Dev.	25th perc.	50th perc.	75th perc.
$C'_2$	7.5	5.66 (0.21)	1.87 (0.17)	5 (0.49)	6 (0.44)	6 (0.48)
$C'_3$	7.5	5.79 (0.26)	2.29 (0.2)	5 (0.3)	5 (0.5)	7 (0.53)
$R_1 \& C'_2$	3.5	4.77 (0.21)	1.81 (0.19)	4 (0.42)	5 (0.45)	5 (0.5)
$R_1 \& C'_3$	3.5	4.41 (0.22)	1.91 (0.19)	3 (0.13)	4 (0.44)	5 (0.46)

Figure D.2: Switch Point Summary. The numbers represent the average number of pair (of the ten pairs) in each round at which participants switched from choosing risky payoffs to certain payoffs. Standard errors are shown in the parentheses.